# The Green Solow Model

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Abstract: We demonstrate that a key empirical finding in environmental economics - The Environmental Kuznets Curve - and the core model of modern macroeconomics - the Solow model - are intimately related. Once we amend the Solow model to incorporate technological progress in abatement, the EKC is a necessary by product of convergence to a sustainable growth path. Our amended model, which we dub the "Green Solow", generates an EKC relationship between both the flow of pollution emissions and income per capita, and the stock of environmental quality and income per capita.The resulting EKC may be humped shaped or strictly declining. We explain why current methods for estimating an EKC are likely to fail whenever they fail to account for cross-country heterogeneity in either initial conditions or deep parameters. We then develop an alternative empirical method closely related to tests of income convergence employed in the macro literature. Preliminary tests of the model's predictions are investigated using data from OECD countries.

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# 1 Introduction

The goal of this paper is to provide a cohesive theoretical explanation for three puzzling features of the pollution and income per capita data. To do so we introduce the reader to a very simple growth model closely related to the one-sector Solow model. We show how this amended model generates predictions closely in line with evidence on emissions, emission intensities and pollution abatement costs. We then use this model to derive a simple estimating equation linking a measure of emissions growth to initial emission levels and other controls drawn from theory. Preliminary tests of the model are encouraging and well in accord with the theoretical predictions of the Green Solow model.

Our work is related to recent attempts to explain the Environmental Kuznets Curve (thereafter EKC) but differs from other contributions in two important ways. First, we attempt to fit more features of the data than just the EKC and employ data on both pollution abatement costs and emission intensities to identify key features of the data that are largely inconsistent with existing theories. Second, we derive an estimating equation directly from our theory. By doing so we provide the first rigorously developed link between theory and empirical work in this area.

The EKC has captured the attention of policymakers, theorists and empirical researchers alike since its discovery in the early 1990s. The theory literature has from the start focussed on developing models that replicate the inverted U shaped relationship. Prominent explanations are threshold effects in abatement that delay the onset of policy, income driven policy changes that get stronger with income growth, structural change towards a service based economy, and increasing returns to abatement that drive down costs of pollution control.[1]

While each of these explanations succeeds in predicting a EKC, they are typically less successful at matching other features of the income and pollution data. One key feature of this data concerns the timing of pollution reductions. Models of threshold effects predict no pollution policy at all over some initial period followed by a period of active regulation.[2] When policy is inactive, emissions are produced lock step with output. When policy is active emissions per unit of output fall as do aggregate emissions. As a result, the decline in the emissions to output ratio occurs simultaneously with the reduction in aggregate pollution levels. This temporal correlation is however strongly contradicted by the data.

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[1] See for example Stokey (1998), Andreoni et al. (2001), and Lopez (1994) for original contributions. A review of the competing explanations appears in Chapter 2 of Copeland and Taylor (2003).

[2] This is for example the exact prediction of both Stokey (1998) and Brock and Taylor (2003a).

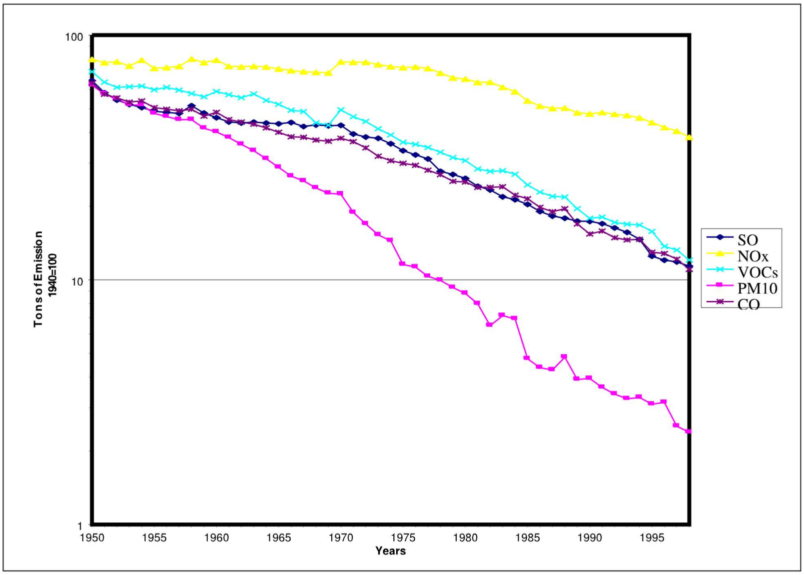

Figure 1: Emission Intensities

In Figure 1 we plot US data giving emissions per dollar of (real) GDP over the 1950 to 2001 period.[3] For ease of reading we have adopted a log scale. We plot emission intensities for sulfur dioxide, nitrogen dioxide, particulate matter, carbon monoxide, and volatile organic compounds. There are two features of note in the figure. The first is simply that the emission to output ratio is in decline from the start of the period in 1950. The second is that (given the log scale for emissions per dollar of output) the percentage rate of decline has been roughly constant over the fifty-year period (although it does vary across pollutants).

In Figure 2 we plot the corresponding emission levels for these same pollutants over the same time period. Figure 2 shows a general tendency for emissions to at first rise and then fall over time. Since the US exhibited trend growth in real income per capita of approximately 2% a year over this period, the time scale in the figure could just as well be replaced by income per capita, and hence it offers a strong confirmation of the EKC as found, for example, by Grossman and Krueger (1994,1995). The EKC pattern is visible in the data for all pollutants except nitrogen oxides that may at present be approaching a peak, and particulates which peaked before the sample period.

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[3] Data on US emissions of the criteria pollutants graphed in Figures 1 and 2 come from the US E.P.A. The long series of historical data presented in the figures is taken from the EPA's 1998 report National Pollution Emission Trends, at http://www.epa.gov/ttn/chief/trends/trends.98. Because prior to 1985 fugitive dust sources and other miscellaneous emissions were not included in PM10 we have removed these components to make the data comparable over time.

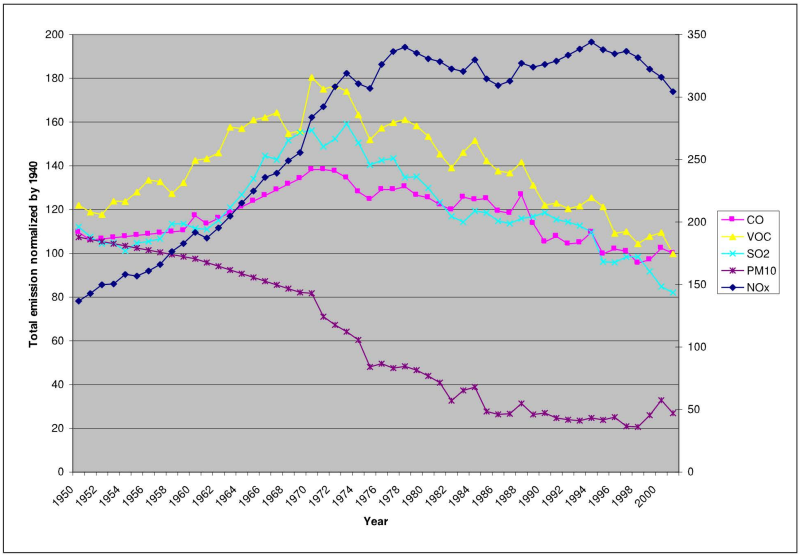

Figure 2: Emission Levels

It is clear however that the reduction in emission intensities shown in Figure 1 precede the peak level of pollutants in Figure 2 by 25 years for sulfur dioxide, carbon monoxide, and volatile organic compounds. Particulates have however been falling throughout, but the peak for nitrogen oxides occurs approximately 50 years after their emissions per unit output started to decline.

If we take the early 1970's as the start of serious pollution regulation, then threshold theories predict an unchanged and therefore horizontal line for the emissions to output ratio until the mid 1970s, and then a steep decline that forces aggregate emissions downward. This is not what Figure 1 and 2 show. The peaks in these pollution profiles – to the extent that they have peaked at all – occur much too late relative to the decline in emission intensities.

A second feature of the data that is difficult to reconcile with many theories is the magnitude of pollution abatement costs. Theories that rely on rising incomes driving down emissions via tighter pollution policy must square very large reductions in emissions with very small pollution abatement costs. For example in Figure 2, sulfur dioxide emissions peaked in 1973 at approximately 32,000 tons and fell almost in half to approximately 17,000 tons in 2001. Correspondingly large changes in emissions per unit output also occurred. But over much of this period, pollution abatement costs as a fraction of GDP or manufacturing value-added, remained both small and without much of a positive trend. Theories that rely on tightening environmental policy predict ever increasing costs of abatement, since emissions per unit of output must fall faster than aggregate output to hold pollution in check. In a world without technological progress in abatement, this requires larger and larger investments in pollution control.[4]

In Figure 3 we plot business expenditures on pollution abatement costs per dollar of GDP over the period 1972-1994. These twenty-two years are the only time period where data is available. As shown, pollution abatement costs as a fraction of GDP rise quite rapidly until 1975 and then remain relatively constant. As a fraction of overall output, these costs are small. Generating a similar plot for costs as a fraction of manufacturing value-added produces similar results. Alternatively, if we consider pollution abatement costs specifically directed to the six criteria air pollutants and scale this by real US output, the ratio is then incredibly small – approximately one half of one percent of GDP - and has remained so for over twenty years (See Vogan (1996)). There is of course considerable controversy over whether these figures represent the full cost of environmental regulation, and they necessarily ignore the significant abatement done prior to the 1970s by cities, utilities, and businesses.[5] Nevertheless, data from other countries supports our general conclusion that pollution abatement costs are a small fraction of GDP and show at best a slight upward trend.[6]

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[4] Stokey (1998), Aghion and Howitt (1998) and others adopt an abatement function relating emissions per unit final output, $E/Y$, to the share of productive factors used in abatement $\theta$, as follows: $E/Y = (1-\theta)^{\beta}$, $\beta > 0$. Copeland and Taylor (2003, Chapter 2) show this relationship arises from an assumption on joint production and constant returns to scale in abatement. For emissions to decline while final output $Y$ grows, $E/Y$ must fall and this implies $\theta$ must approach 1. That is, the share of the economy's resources dedicated to abatement must rise along the model's balanced growth path and approach one in the limit. The interested reader can verify this by making the translation into Stokey's notation by setting $1-\theta = z$, and interpreting the gap between Stokey's potential and actual output as the output used in abatement.

[5] For an illuminating historical account of pollution regulation in the US from 1940 to 1970 see Dewey (2000). Dewey details the efforts at pollution control in major US cities such as New York, St. Louis, Pittsburgh and Los Angeles. The analysis shows serious pollution regulation is not a post 1970s phenomena.

[6] US Data shown in Figure 3 is taken from Vogan (1996). See Table 3, section 4 for International data on pollution abatement costs, and our data appendix for a summary of the measures used in our empirical work.

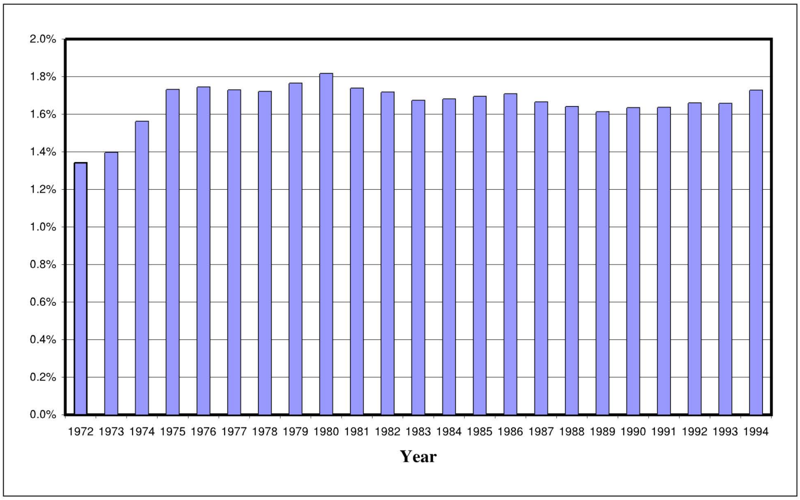

Figure 3: US Abatement Costs/GDP

While income effect theories often point to the creation of the EPA in the 1970s and more activist environmental policy, we should again return to Figure 1 and note that the trend in emissions to output was already declining and strongly so prior to 1970. Therefore the advent of more activist federal policy in the 1970s can only be a contributor to processes already at play in the 1950s.

Theories relying on strong compositional shifts or increasing returns also have difficulty matching these data. Changes in the composition of output towards less pollution intensive goods can lower emissions in the medium term, but in the long term reductions can only occur if emissions per unit of output in the cleanest of goods falls. This of course places us back where we started, asking how to lower emissions per unit output without ever rising costs. Moreover, empirical work has found a changing composition of output plays at most a bit part in the reductions we have observed (Selden et al. (1997), Bruvoll et al. (2003)).

And while increasing returns to abatement may be important in some industries and for some processes, a large portion of emissions come from small diffuse sources such as cars, houses and individual consumptive activity. In each of these cases, increasing returns to abatement seems unlikely. Increasing returns also presents strong incentives for mergers and natural monopoly and unless we bound the strength of increasing returns carefully, IRS models predict negative pollution emissions at large levels of output.[7]

To us the pollution data and the related empirical work on the EKC present three puzzles that need to be resolved by any successful theory.

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[7] The simplest version of Andreoni and Levinson's theory of increasing returns to abatement has the property that pollution becomes negative for some large, but finite level of output. This feature poses problems in dynamic models where output grows exponentially.

The first puzzle is how do we square the very large reductions in emission intensities shown in Figure 1 with the relatively small pollution abatement costs shown in Figure 3?

The second puzzle is the EKC: what is responsible for the shape of the pollution profiles in Figure 2?

The third puzzle comes from the empirical literature itself. What explains the current disconnect between the evidence for the EKC present in plots of raw data like Figure 2, and the difficulty empirical researchers have in estimating EKC relationships? It is now well known that empirical estimates from EKC style regressions can vary greatly with the sample used and estimation procedure. How do we make sense of the finding of an EKC in raw country level data as shown in Figure 2, and the fragile cross-country empirical results that are now commonplace to the literature?

In this paper we show that the Green Solow model provides a very simple explanation for all three puzzles. Our explanation starts with the observations in Figure 1 and 3. We square the rapidly declining emission intensities shown in Figure 1 with the constant pollution abatement costs in Figure 3 by assuming ongoing technological progress in abatement. To capture this possibility we introduce exogenous technological progress into a standard abatement function and then couple this abatement function with a standard fixed savings rate Solow model. The resulting "Green Solow model" then generates a pattern of incomes per capita and pollution consistent with Figure 2; i.e. it generates an EKC.

The logic is simple: Ongoing technological progress in abatement drives emissions per unit of output downward at a constant rate both in and out of steady state (as in Figure 1). Initially the Solow model's fast initial growth overwhelms progress in abatement to produce a period of initially rising emission levels. Aggregate emissions rise even though emissions per unit of output are falling (recall Figure 2). Technological progress in abatement however eventually overwhelms the slowing growth of output as the economy approaches its balanced growth path. Aggregate emissions start to decline while emissions per unit of output continue their fall. Throughout the model's measure of pollution abatement costs as a fraction of GDP is constant (recall Figure 3). We offer these features of the model as potential explanations for the first two puzzles in the data.

Another model prediction is that the path for emissions, peak level of emissions, and income per capita at peak emissions will typically be country specific. Even countries that share identical parameter values will exhibit different EKC patterns if they differ in initial conditions. Additional cross-country heterogeneity is introduced by differences in savings rates, population growth rates or abatement intensities. Failing to account for this heterogeneity could be responsible for the failed empirical tests, and the sensitivity of estimates to the sample. We take this feature of the model as a potential explanation for the third puzzle - the current disconnect between the plots of raw data showing an EKC within countries, and the fragility of cross country empirical results. While much of current empirical work on the EKC includes controls for cross-country heterogeneity these controls are typically level variables such as population density, openness to trade, or measures of democracy and not the rates of change variables suggested by our analysis.

Finally to complete our argument, we provide empirical evidence in support of our approach from sources outside the dataset we sought to explain. Since our theoretical work shows that EKC profiles are not unique we focus our attention on a model prediction that holds more generally: convergence in a measure of emissions per capita. By borrowing from techniques used in the macro literature on income convergence we derive a simple linear estimating equation linking growth in emissions per capita over a fixed time period to emissions per capita in an initial period and a limited set of controls. These controls include typical Solow type regressors such as population growth and the savings rate, but also include a measure of pollution abatement costs and a proxy for technological progress in abatement. To demonstrate the potential usefulness of our approach we estimate our specification on OECD data. The results are encouraging.

Not surprisingly, the Green Solow model bears a family resemblance to many other contributions in the literature given its close connection to Solow (1956). It is similar in purpose to that of Stokey (1998) but differs because Stokey does not consider technological progress in abatement. It is related to the new growth theory model of Bovenberg and Smulders (1995) because these authors allow for "pollution augmenting technological progress", which is, under certain circumstances, equivalent to our technological progress in abatement. The focus of their work is however very different from ours. It is perhaps most closely related to our own earlier work (Brock and Taylor (2003a)) where we tried to match data on pollution abatement costs, the EKC, and emission intensities within a modified AK model with ongoing technological progress in both goods and abatement production. While our earlier work was successful in some respects, like other models with threshold effects it failed to predict the steady fall in emission to output ratios prior to peak pollution levels. And while this earlier work contained a prediction regarding convergence in emission levels, this prediction did not follow from the neoclassical forces we highlight here. This paper grew out of our earlier attempts to match key features of the pollution and income per capita data within the simplest model possible. Our work also owes much to previous work in macroeconomics on conditional and absolute convergence; in particular Barro (1991) and Barro and Sala-i-Martin (1992).

The rest of the paper proceeds as follows. Section 2 sets up the basic model and develops three propositions concerning its behavior. In section 3 we derive an estimating equation from the model and present a preliminary empirical implementation using CO2 data from the OECD. Section 4 contains a discussion of our assumptions and offers some international evidence. To make our points clear we develop the model under the assumption that both savings rates and abatement intensities are fixed over time. The appendix contains all proofs and lengthy calculations.

## 2 The Model

We develop an augmented Solow model where exogenous technological progress in both goods production and abatement leads to continual growth with rising environmental quality. We present the simplest specification where both savings and abatement choices are exogenously set. The fixed savings rate assumption is commonly used in the Solow model and is often innocuous; the assumption of a fixed abatement intensity helps us demonstrate how changes in the intensity of abatement need not play any role in generating an Environmental Kuznets Curve. Together they render the model simple and tractable.

Consider the standard one sector Solow model with a fixed savings rate $s$. Output is produced via a constant returns to scale and strictly concave production function taking effective labor and capital to produce output, $Y$. Capital accumulates via savings and depreciates at rate $\delta$. We assume the rate of labor augmenting technological progress is given by $g$. All this implies:

$$\begin{aligned} Y &= F(K, BL), \overset{\bullet}{K} = sY - \delta K \\ \overset{\bullet}{L} &= nL, \overset{\bullet}{B} = gB \end{aligned} \tag{1}$$

where $B$ represents labor augmenting technological progress and $n$ is population growth.

To model the impact of pollution we follow Copeland and Taylor (1994) by assuming every unit of economic activity, $F$, generates $\Omega$ units of pollution as a joint product of output.[8] The amount of pollution released into the atmosphere may differ from the amount produced if there is abatement. We assume abatement is a constant returns to scale activity and write the amount of pollution abated as an increasing and strictly concave function of the total scale of economic activity, $F$, and the economy's efforts at abatement, $F^A$. If

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[8] This approach has been subsequently employed by many authors (Stokey (1998), Aghion and Howitt (1998), etc.). In these other papers, $\Omega$ is taken as constant over time and by choice of units set to one. Some authors who adopt this approach refer to the firms or planners problem as one of choosing across dirty or clean technologies rather than less or more abatement. Copeland and Taylor (2003, chapter 2) provides background and shows the two approaches are identical.

abatement at level $A$, removes the $\Omega A$ units of pollution from the total created, we have:

$$\begin{aligned} \textit{pollution emitted} &= \textit{pollution created} - \textit{pollution abated} \qquad (2) \\ E &= \Omega F - \Omega A(F, F^A) \\ E &= \Omega F\left[1 - A(1, F^A/F)\right] \\ E &= \Omega F a(\theta), \\ \textit{where } a(\theta) &\equiv \left[1 - A(1, F^A/F)\right] \textit{ and } \theta = F^A/F \end{aligned}$$

where the third line follows from the linear homogeneity of A, and the fourth by the definition of $\theta$ as the fraction of economic activity dedicated to abatement. We assume the intensive abatement function satisfies $a(0) = 1$ and note $a'(\theta) < 0$ and $a''(\theta) > 0$ by concavity. Abatement has a positive but diminishing marginal impact on pollution reduction. In some cases we will adopt the specific form $a(\theta) = (1 - \theta)^{\epsilon}$ where $\epsilon > 1$.

The relationship in 2 requires several comments. The first is simply that 2 shows emissions are determined by the scale of economic activity $F$, and the techniques of production as captured by $\Omega a(\theta)$. Techniques can be influenced by changes in the intensity of abatement, $\theta$, or by technological progress that lowers the parameter $\Omega$ over time. Since $F^A$ is included in $F$, even the activity of abatement itself pollutes. Second, abatement uses factors in the same proportion as does final output hence we can think of the fraction $\theta$ of capital and effective labor being allocated directly to abatement with the remaining fraction $(1 - \theta)$ available for production of consumption or investment goods. Finally, it is important to note that a fixed abatement intensity, $\theta$, does not correspond to a situation of static or non-existent environmental policy. We show in the appendix that $\theta$ remains constant over time if governments raise technology standards slowly over time. Our reading of environmental history suggests this may be a reasonably accurate characterization of slowly evolving technology standards imposed via command and control.

To combine our assumptions on pollution in 2 with the Solow model, we note that once we take abatement into account, output available for consumption or investment $Y$, then becomes $Y = [1 - \theta]F$.

Since we wish to generate predictions on both environmental quality and emissions we must adopt some assumption concerning natural regeneration. The simplest form has exponential dissipation of pollution so that the stock of pollution X is related to the flow of emissions E according to:

$$\overset{\bullet}{X} = E - \eta X \qquad (3)$$

where $\eta > 0$ is the natural rate of regeneration and $X = 0$ represents a pristine environment with a zero pollution stock.

Finally, to match the Solow model's exogenous technological progress in goods production raising effective labor at rate $g$, we assume exogenous technological progress in abatement lowering $\Omega$ at rate $g^A > 0$. Putting these assumptions together and transforming our measures of output, capital and pollution into intensive units, the Green Solow model becomes:

$$\begin{aligned} y &= f(k)[1-\theta] && (4) \\ \overset{\bullet}{k} &= sf(k)[1-\theta] - [\delta + n + g]k && (5) \\ e &= f(k)\Omega a(\theta) && (6) \end{aligned}$$

where $k = K/BL$, $y = Y/BL, e = E/BL$ and $f(k) = F(k, 1)$.

## 2.1 Balanced growth path

Assume the Inada conditions hold for $F$, then with $\theta$ fixed it is immediate that starting from any $k(0) > 0$, the economy converges to a unique $k^*$ just as in the Solow model. As the economy approaches its balanced growth path aggregate output, consumption and capital all grow at rate $g+n$ while their corresponding per capita magnitudes grow at rate $g$. Using standard notation for growth in per capita magnitudes, along the balanced growth path we must have $g_y = g_k = g_c = g > 0$. A potentially worsening environment however threatens this happy existence. Since $k$ approaches the constant $k^*$ along the balanced growth path we can infer from 6 that the growth rate of aggregate emissions along the balanced growth path, $G_E$ , can be positive or negative:

$$G_E = g + n - g_A \qquad (7)$$

The first two terms in 7 represent the scale effect of growth on emissions since aggregate output grows at rate $g+n$ along the balanced growth path. The second term is a technique effect created by technological progress in abatement. Using this information and referring to 3 it is easy to see that constant growth in $X$ along the balanced growth path occurs when $G_X = G_E$.

Define sustainable growth as a balanced growth path generating rising consumption per capita and an improving environment. Sustainable growth is guaranteed by:

$$g > 0 \; and \; g_A > g + n \qquad (8)$$

Technological progress in goods production is necessary to generate per capita income growth. Technological progress in abatement must exceed growth in aggregate output in order for pollution to fall and the environment to improve.

## 2.2 Green Solow and the EKC

The Green Solow model, although simple, generates a very suggestive explanation for much of the empirical evidence relating income levels to environmental quality. Despite the fact that the intensity of abatement is fixed, there are no composition effects in our one good framework, and no political economy or intergenerational conflicts to resolve, the Green Solow model produces a path for income per capita and environmental quality that traces out an Environmental Kuznets Curve. This is true whether we measure environmental quality via our stock variable $X$ or the flow of emissions $E$. This result is shown in Figure 4.[9]

In Figure 4 we present the trajectories for two economies that are identical in all respects except for their allocation to abatement $\theta$. We plot both emissions $E$ and the pollution stock $X$. Each economy starts from an initially pristine environment and a small initial capital stock, $k(0) > 0$. One economy allocates 5% of its output to abatement which we refer to as the strong abatement case; the other economy is the weak abatement case as it allocates only .5% of its output to abatement. Parameters were chosen for the purposes of illustration. We have taken $f(k)$ to be Cobb-Douglas with a capital share of .35. Per capita income grows at 1.5% along the balanced growth path, the population grows at 1% and the abatement technology improves at 3%. These parameters ensure sustainable growth is possible. The savings rate is 25%, depreciation is 3.5%, regeneration is set at .03 implying a 3% rate of dissipation of X per unit time.

As shown, the environment at first worsens with both X and E rising. After approximately 40 years emissions start to fall. After approximately 90 years the pollution stock $X$, starts to fall and the economy converges on its balanced growth path. Using 7 we know that along the balanced growth path emissions fall at .5% per year, which is close to what the simulation delivers in its last periods. Outside of the balanced growth path, emissions growth is of course positive for a long period of time.

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[9] During the final writing of this paper we discovered that Xepapadeas (2003) also notes that technological progress in abatement can generate an EKC pattern. His discussion is brief and appears in a review article as does our first discussion of Green Solow in Brock and Taylor (2003b).

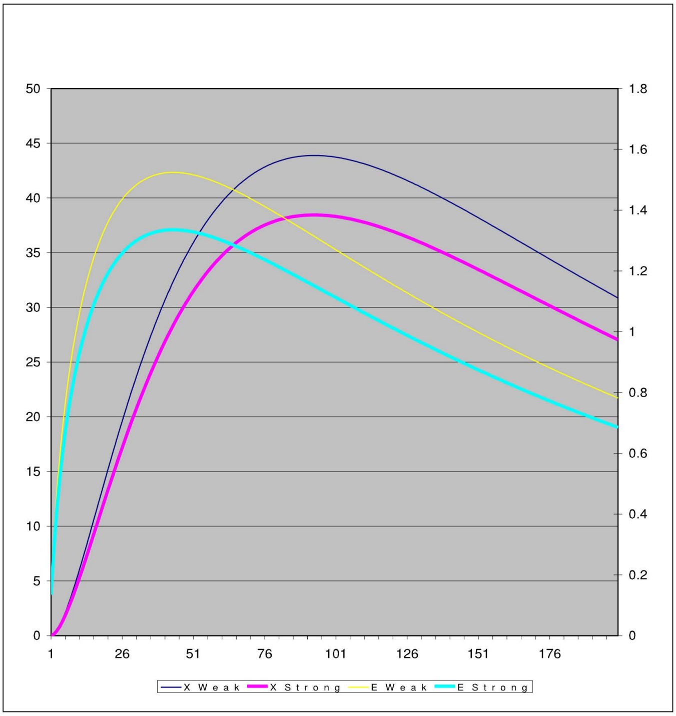

Figure 4: The EKC

The result shown in Figure 4 follows for very simple reasons. The convergence properties of the Solow model imply that output growth is at first very rapid but slows as $k$ approaches its balanced growth path level $k^*$. Pollution emissions grow quickly at first but slower later. Both during the transition phase and beyond, emissions per unit output are falling at the constant rate $g_A$ because of technological progress in abatement. This works to drive emissions downward. Finally, we have assumed growth is sustainable in the long run so that $g_A > g+n$. It is then immediate that the typical convergence properties of the Solow model ensure that rapid growth in output will first overwhelm falling emissions per unit output when the economy is far from its balanced growth path, but growth in output will in turn be overwhelmed by technological progress in abatement sometime before the economy enters its balanced growth path. The interplay of technological progress and diminishing returns generates an EKC.

Outside observers may interpret the correlation between emission reduction and income growth in a variety of ways. One interpretation could be that environmental policy has finally come of age and is now aggressive enough to cause emission levels to fall. Another is that the slowdown in output growth is caused by the tightening environmental policy that is also driving emissions downward. Both of these interpretations are wrong in the context of the Green Solow model. The decline in emissions is not reflective of a new and invigorated environmental policy since $\theta$ is constant over time. And the slowdown in growth is caused by diminishing returns not environmental policy. In fact, the slowdown in growth is the cause of emission decline - not the reverse. While it is quite natural to link a turning point in emissions with a discrete change in circumstances, the model shows that the turning point may instead reflect a more subtle weighing of various forces long at work in the economy.

In generating this result we have of course assumed the fraction of aggregate resources allocated to abatement is roughly constant - recall Figure 3 - and we have assumed technological progress in abatement works to lower emissions per unit output continuously - recall Figure 1. In fact, Green Solow equates the slope of log $E/Y$ shown in Figure 1 to $g_A$ which we have assumed is constant over time. Since the model predicts that emissions per unit of output fall at a constant rate both during the transition period *and* along the balanced growth path, emissions per unit of output are falling long before emissions or the pollution stock peaks. It is tempting therefore to construct the model's analog to the emission intensities graphed in Figure 1 and pollution levels graphed in Figure 2. We construct such a graph for the strong abatement case and present it as Figure 5. The match with the earlier figures is striking.

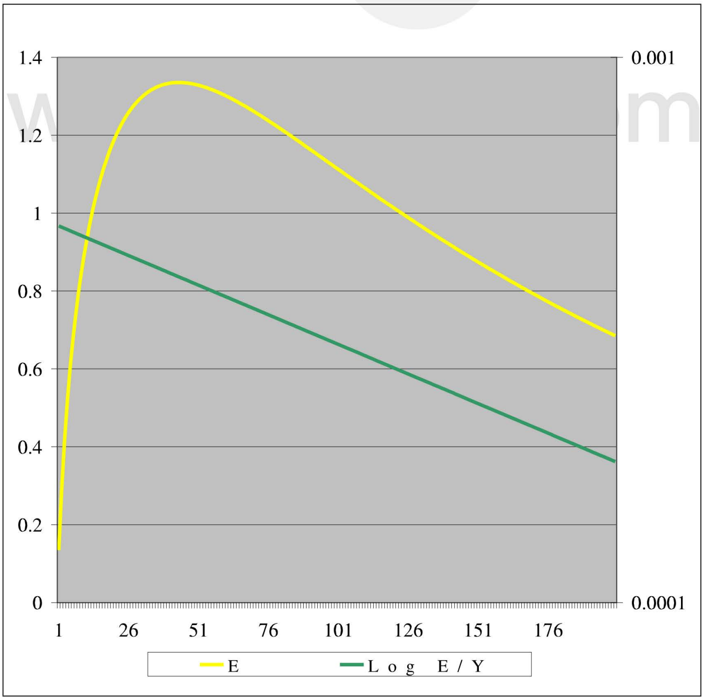

Figure 5: Matching E and E/Y

Thus far we have illustrated the properties of Green Solow by simulation. To investigate how general these results are we need to solve for the transition dynamics explicitly.

## 2.3 Diminishing Returns and the Dynamics of Transition

We examine the transitional dynamics with the aid of two diagrams. The first plots the growth rate of emissions and capital against capital per effective labor and is very similar to graphical representations of the Solow model. The second follows from the first and plots the level of emissions as a function of capital per effective worker and is very similar to representations of the EKC. To start we need to develop a differential equation for emissions. To do so write emissions at any time t as:

$$E = B(0)L(0)\Omega(0)a(\theta)\exp[G_E t]k^{\alpha} \tag{9}$$

where $B(0)$, $L(0)$, and $\Omega(0)$ are initial conditions, and $G_E$ was given earlier. Differentiate with respect to time to obtain the growth rate of emissions:

$$\frac{\dot{E}}{E} = G_E + \alpha\frac{\dot{k}}{k} \tag{10}$$

where we note the rate of change of capital per effective worker is simply:

$$\frac{\dot{k}}{k} = sk^{\alpha-1}(1-\theta) - (\delta+n+g) \tag{11}$$

Using these two expressions we now depict the dynamics in the two panels of Figure 6.

In the top panel of Figure 6 we plot the rates of change of ($\alpha$ times) capital per effective worker $\alpha\dot{k}/k$ and aggregate emissions $\dot{E}/E$ on the vertical axis against capital per effective worker $k$ on the horizontal. In drawing the figure we have implicitly assumed growth is sustainable. We refer to the negatively sloped line as the savings locus since it is given by $\alpha sk^{\alpha-1}[1-\theta]$ and shifts with the savings rate $s$. The savings locus starts at plus infinity and approaches zero as $k$ grows large; therefore, it must intersect the two horizontal lines at points T and B as shown. From 11 it is clear that the vertical distance between $\alpha sk^{\alpha-1}[1-\theta]$ and the horizontal line with height $\alpha[\delta+n+g]$ is just $\alpha$ times the growth rate of capital per effective worker or $\alpha\dot{k}/k$. Capital per effective worker is rising at all points to the left of B and falling at all points to the right. As is well known, the intersection at point B gives us the steady state capital per effective worker $k^*$. Growth is most rapid for small $k$ and falls as $k$ approaches $k^*$. When the economy enters its balanced growth path, $\dot{k}$ is zero and the economy's aggregate output and capital grow at rate $g+n$.

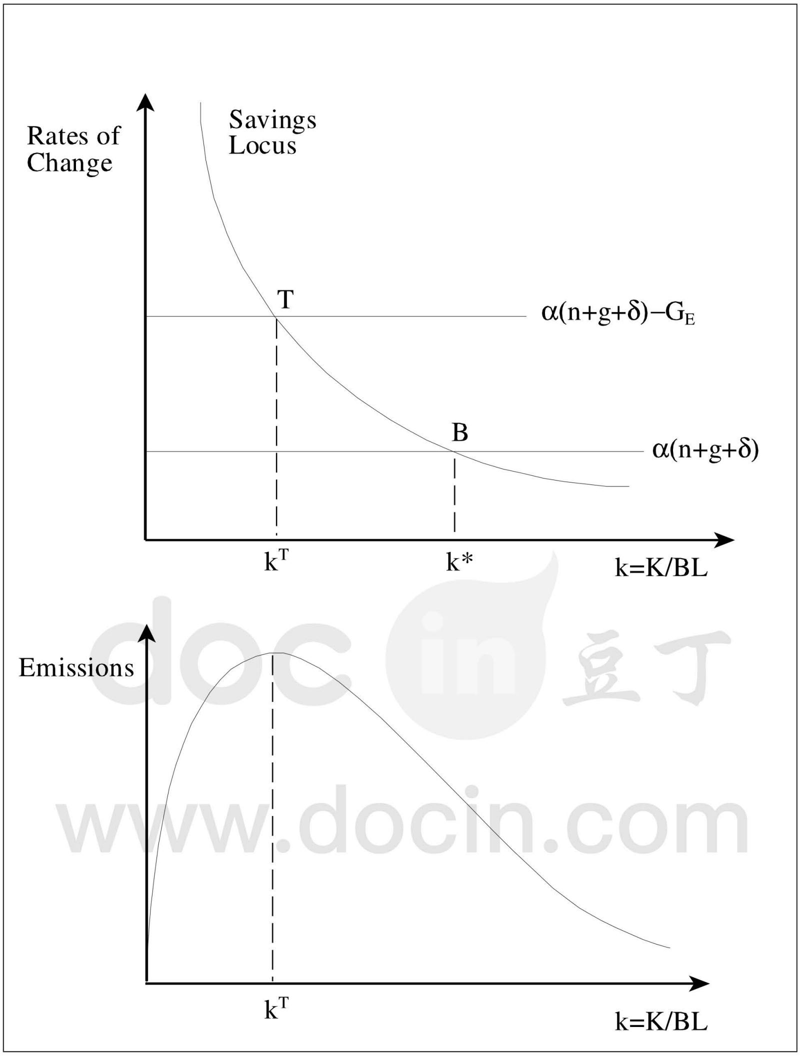

Figure 6: The Green Solow Model

To determine the time path for emissions recall that $G_E$ is constant, and therefore from 10 we conclude that the growth rate of emissions inherits most of the properties of the growth rate of capital. Most importantly, the growth rate of emissions is very rapid for small $k$ and falls monotonically as the economy approaches its balanced growth path. We will exploit this property later when we derive an estimating equation predicting the convergence in emissions across countries. But for now it is important to recognize that the growth rate of emissions falls regardless of whether growth is sustainable or not.

To determine the peak level of emissions we use 10 and 11. By construction the vertical distance between the savings locus $\alpha s k^{\alpha-1}[1-\theta]$ and the horizontal line with height

$\alpha[\delta + n + g] - G_E$ equals the percentage rate of change of emissions or $\overset{\bullet}{E}/E$. Therefore, at point T the growth rate of emissions is zero: $\overset{\bullet}{E} = 0$. Point T represents the turning point in emissions as shown in the bottom panel of Figure 6. Under the assumption that growth is sustainable, $G_E < 0$, and point T lies to the left of B; when growth is not sustainable $G_E > 0$ and T lies to the right of B.

The figure illustrates several features of the model. It shows that if an economy's growth path is unsustainable, then emissions will grow ad infinitum even as the economy approaches its balanced growth path. But even in the unsustainable case the growth rate of emissions falls along the transition path until it approaches its balanced growth path rate from above. If growth is sustainable then T lies to the left of B and the time profile for emission levels depends on the location of $k(0)$ relative to point T. If an economy starts with a small initial capital stock then emissions at first rise and then fall as development proceeds: i.e. we obtain an EKC profile for emissions. If initial capital is larger it is possible that the level of emissions falls monotonically as the economy moves towards its sustainable growth path. It is important to note while the level of emissions may rise and then fall over time, the growth rate of emissions is monotonically declining. This is apparent because emissions growth is rapid for countries a long way from point B, and slower for those near B regardless of the location of T. Finally when emissions peak depends on the relationship between points T and B. For example, if $-G_E$ is small, then T and B differ very little and emissions will only peak as the economy approaches its balanced growth path which may of course take a very long time. Since these are key results, we record them as a proposition.

**Proposition 1** *If growth is sustainable and $k^T > k(0)$, then the growth rate of emissions is at first positive but turns negative in finite time. If growth is sustainable and $k(0) > k^T$, then the growth rate of emissions is negative for all $t$. If growth is unsustainable, then emissions growth declines with time but remains positive for all $t$.*

Proof: See Appendix

Proposition 1 tells us about the shape of the emissions and income profile but says very little about the level of emissions and income per capita at the turning point. Although the model is simple, it can be deceptive in this regard. For example, it is a short step from knowing that $k^T$ is unique to an assumption that income per capita at the turning point is unique. Similarly, it is easy to assume that the path for income growth and emissions is the same for countries sharing savings rates, population growth rates, etc. Both of these conjectures are wrong: although $k^T$ is unique, the associated income per capita and emissions level at $k^T$ are not.

**Proposition 2** *Economies with identical parameter values but different initial conditions produce different income per capita and emission profiles over time. The peak level of emissions and the level of income per capita associated with peak emissions are not unique.*

Proof: in text.

The intuition for this result is straightforward. The peak level of emissions is reached when the rate of emissions growth created by output growth equals the rate of technological progress in abatement. This occurs at a unique $k^T$. Take two economies with the same physical capital and assume both economies are at $k^T$. While these economies must have the same effective labor force at this point, one economy could have a highly efficient but small working population while the other had a less efficient but more numerous labor force. Clearly income per capita differ in these two economies even though each is at $k^T$.

The nonuniqueness of emissions follows for related reasons. An economy that is larger has greater emissions everywhere even though it may have the same capital per effective worker along the transition path as some hypothetical smaller economy (see 9). Less transparently an economy with a inferior abatement technology (a higher $\Omega(0)$) will have a higher emissions per unit of output leading to a difference in peak emissions at the turning point and elsewhere.

These examples highlight an important point brought out by the Green Solow model. The current literature has tended to focus our attention on level variables - specifically the level of pollution against the level of income per capita. Even the "control variables" added to EKC regressions are often level variables such as population density, openness to trade, measures of democracy or deposits of coal. The Green Solow model refocuses our attention on growth rates since it is the equality of two growth rates that determines the turning point in emissions. By doing so it shows how looking at the levels of variables can be misleading.

The non uniqueness of peak income and emission levels, offers a potential explanation for the contradictory and sometimes erratic empirical results found in the EKC literature. It is now well known that the shape of the estimated EKC can differ quite widely when researchers vary the time period of analysis, the sample of countries, the pollutant, or even the data source. For example, Harbaugh et al. (2002) reconsider Grossman and Krueger's specification and find little support for an EKC using newer updated data. Stern and Common (2001) employ a larger and different sulfur dioxide dataset and find no EKC. And the literature reviews by both Barbier (1997) and Stern (2003) note that published work differs greatly in the estimated turning points for the EKC, the standard errors on turning points are often very large, and empirical results differ widely across pollutants and countries. At the same time, plots of raw pollution data for the US and other countries often present a dramatic confirmation of the EKC.[10]

Proposition 2 offers a simple explanation for the seeming inconsistency between country level data and cross-country empirical results. If EKC profiles for even very similar countries are not unique because of differences in initial conditions, then unobserved heterogeneity is surely a problem. Unobserved heterogeneity could then account for the large standard errors on turning points and the sensitivity of results to the sample. While in theory conditioning on country characteristics could eliminate the problem of unobserved heterogeneity, existing work has focussed on additional controls that are level variables and not the rate of change variables suggested by our theory.

To be more precise concerning peak emission and income levels write income per capita at any time t as:

$$y^c(t) = k(t)^\alpha B(0)[1-\theta]\exp[gt] \tag{12}$$

which is a function of $k(t)$, time, abatement and the initial condition $B(0)$. At the turning point, emissions growth is zero and solving for the $k^T$ identified in Figure 6 yields:

$$k^T = \left[\frac{s(1-\theta)}{n+g+\delta-G_E/\alpha}\right]^{1/(1-\alpha)} \tag{13}$$

To solve for the time - call it $t = T$ - at which the economy's capital per effective worker reaches $k^T$ solve the differential equation for $k(t)$ to find:

$$\begin{aligned} k(t) &= \left[k^{*(1-\alpha)}(1-\exp[-\lambda t]) + k(0)^{(1-\alpha)}\exp[-\lambda t]\right]^{1/(1-\alpha)} && (14) \\ k^* &= \left[\frac{s(1-\theta)}{n+g+\delta}\right]^{1/(1-\alpha)} && (15) \end{aligned}$$

As expected $k(t)$ is an exponentially weighted average of initial capital per worker $k(0)$ and its balanced growth path level $k^*$ where the weight given to initial versus final positions is determined by the speed of adjustment in the Solow model $\lambda = [1-\alpha][n+g+\delta]$. We can now set $k(t)$ equal to $k^T$ yielding an implicit equation for the time it takes to reach the peak level of emissions. $T$ is defined by:

$$T : k^T = \left[k^{*(1-\alpha)}(1-\exp[-\lambda T]) + k(0)^{(1-\alpha)}\exp[-\lambda T]\right]^{1/(1-\alpha)} \tag{16}$$

Note that $k(0) = K(0)/B(0)L(0)$. Income per capita at the peak is found by evaluating 12 using $T$ from 16 and subbing in for $k^T$ using 13. Peak emission levels follow similarly.

To verify that income per capita is not unique at the peak level of emissions note that 16

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[10] For international evidence see Table 3 in Section 4.

shows us that $T$ is independent of variations in initial conditions that leave $k(0)$ unchanged. At the same time, from 12 it is apparent that any variation in $B(0)$ alters income per capita directly even if $k(0)$ and $T$ are left unchanged. To see that emissions are not unique substitute 13 into 9 and again consider variations in initial condition $B(0)$ leaving $k(0)$ unchanged. These variations have no effect on the time to peak emissions, but will affect emissions directly by altering effective labor. Note that $\Omega(0)$ plays no role in determining $k^T$ or $T$; hence variations in it alter the peak level of emissions directly via 9. Even if we corrected for country size by measuring emissions per person, countries with a higher emissions per unit of output at time zero will have greater emissions as well.

Despite these indeterminacies it remains true that every economy will follow an EKC pattern as described in Proposition 1. Since empirical work regresses emissions on income per capita and not time as we have here, it is useful to make the connection between our theory and the existing empirical work precise. To do so use 14 in 9 to find:

$$\begin{aligned} E(t) &= c_0 \exp[G_E t]\left[\left[k^{*(1-\alpha)}(1-\exp[-\lambda t]) + k(0)^{(1-\alpha)}\exp[-\lambda t]\right]^{\alpha/(1-\alpha)}\right] \\ c_0 &= B(0)L(0)\Omega(0)a(\theta) \end{aligned} \qquad (17)$$

But from 14 and 12 it is apparent that $y^c(t)$ is a strictly increasing function of time. We can therefore invert it finding $t = \phi(y^c)$ and substitute for time in 17. This gives us a parametric relationship between aggregate emissions and income per capita. Establishing the properties of this relationship requires further work that we leave to the appendix, but we note here:

**Proposition 3** *There exists a parametric relationship between emissions $E$ and income per capita $y^c$ that we refer to as an EKC. If $k^* > k^T > k(0)$, then emissions first rise and then fall with income per capita. If $k^* > k(0) > k^T$, then emissions fall monotonically with income per capita.*

Proof: See Appendix

Proposition 3 is important in establishing that the Green Solow model reproduces an EKC relating emissions to per capita income. This EKC may take on a typical hump shape or it may be monotonically declining as some authors have found for some pollutants. It is important however to recognize that both income per capita and emissions are both functions of more primitive determinants such as initial conditions, savings rates, etc. Even though an EKC relationship exists in the Green Solow model, strictly speaking there is no causal relationship between income per capita and emission levels. Therefore, the typical processes held responsible for an EKC can be very weak or even non-existent and yet have researchers observe an EKC pattern in the data.

## 2.4 Comparative Steady State Analysis

Most of the empirical exercises investigating the EKC employ cross country data that includes both developed and developing countries and often both democracies and communist states. Clearly these economies differ in much more than just initial conditions, and this heterogeneity may further confound estimation. To investigate how differences in deep parameters affect our results we now consider the impact of changes in savings, abatement and rates of technological progress.

Consider the role of savings. An increase in the savings rate shifts the savings locus rightward raising both T and B in Figure 6. Greater savings raises capital per effective worker in steady state. The turning point for emissions rises because higher savings implies more rapid capital accumulation at each $k$. This in turn means faster output growth and faster emissions growth at any given $k$. The turning point can only be reached when diminishing returns lowers output growth to meet $-g_A$; greater savings makes this task harder and hence $k^T$ rises.

To determine whether economies that save more will reach peak emissions at a higher or lower income per capita write income per capita at the peak as:

$$y^c(T) = [k^T]^\alpha B(0)[1-\theta]\exp[gT]$$

Income per capita at the peak is rising in capital per effective worker at the peak and rising in the calendar time needed to reach the peak. The former determines the capital intensity of the economy, the latter determines how productive labor is when the transition point is reached. We have already shown $k^T$ rises with $s$. To solve for the calendar time to transition, rearrange 16 to find:

$$T = \frac{1}{\lambda}\log\left[\frac{k^{*(1-\alpha)} - k(0)^{1-\alpha}}{k^{*(1-\alpha)} - k^{T(1-\alpha)}}\right] \tag{18}$$

The calendar time needed to reach peak emissions is declining in the convergence speed of the Solow model, $\lambda$, increasing in the gap between initial and final capital per effective worker, and is larger the closer is point T to B. If we substitute for $k^*$and $k^T$ in 18 it is possible to show that an increase in the savings rate raises $T$. Since savings also raises capital per effective worker at the turning point we are done: an economy with a higher savings rate reaches its peak emissions level at a higher income per capita than otherwise.

Differences in abatement intensity have a similar but opposite effect. An increase in abatement lowers $(1-\theta)$ and shifts the savings locus leftward. This reduces both T and B. Since more resources are devoted to abatement and less to savings, larger investments in abatement slow transitory growth and for any given $k$, they imply slower growth in emissions as well. Using 18 we can show that $T$ falls as well. Therefore, peak emissions are reached at a lower level of income per capita when abatement is more aggressive. It is very important to note that emissions start to fall at a lower income per capita not because abatement lowers emissions per se although it does this in the level sense, but because abatement uses up scarce resources that would otherwise have gone to investment. This reduces the rate of growth of output during the transition period. It is the impact of abatement on growth rates during the transition that alters $k^T$. Changes in the abatement intensity have no effect whatsoever on the economy's long run growth or on the long run growth rate of emissions.[11]

Finally consider the impact of changes in technological progress. Start with changes in the rate of progress in abatement, $g_A$. An increase in $g_A$, pushes emissions down faster and shifts the uppermost line in Figure 6 upwards lowering $k^T$. This lowers the growth rate of emissions for any $k$, and will likewise lower the growth rate of emissions in steady state. This change has no effect on the growth rate of output or on $k^*$. Not surprisingly using 18 we find that $T$ is reduced. Putting these results together we find peak emissions are reached at a lower income per capita than otherwise.

Faster technological progress in goods production has a less clear cut effect. An increase in $g$ shifts the uppermost line in Figure 6 downward raising T and the lowermost line upwards lowering B. The time to peak emissions could rise or fall, and hence income per capita at the peak may be higher or lower. All else equal income is higher since capital intensity at the peak has risen, but income may be lower if the calendar time needed to reach the peak is lower than before. A somewhat similar result arises from changes in population growth. Population growth lowers steady state capital per worker and this lowers transitional growth at any given $k$. But population growth raises emissions directly via a scale effect and this raises both emissions growth and the point at which emissions start to fall. Whether this new higher transition point is reached sooner or later in calendar time is indeterminate and hence so too is the associated income per capita.

These results demonstrate that there are three qualitatively different sets of parameters in the model. The first set are parameters (such as initial conditions) that affect emissions and income levels at their peak but have no effect on long run growth rates of emissions or ouput nor any affect on the steady state. A second set of parameters (such as savings rates and abatement intensities) affect both peak emissions and income levels, alter steady state levels and have an impact on transitional growth, but have no impact on long run growth rates. The final set of parameters (such as rates of technological progress or population

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[11] This doesn't mean more abatement has no costs: greater abatement lowers the level of income per capita along the balanced growth path.

growth) alter peak emissions and income, transitional growth and growth along a balanced growth path. In short these results demonstrate that the relationship between income and pollution is exceedingly complex. And therefore it should come as no surprise that there are large standard errors on turning points and fragile coefficient estimates.

To our knowledge no empirical work examining the growth and environment relationship has used as controls savings rates, population growth rates, etc. that would be suggested by our analysis. But even with information on deep parameters we have already shown that since the EKC profile is reliant on initial conditions, estimation problems remain. One alternative that presents itself is to focus on model predictions that are tightly linked to parameters: that is focus on the relationship between the growth rate of emissions along the transition path rather than the parametric relationship between the level of emissions and income.

# 3 An Empirical Implementation

We have demonstrated why current empirical methods may have difficulty in estimating an EKC relationship. The income-emissions profile will differ across countries if they differ in initial conditions or in basic parameters such as savings or population growth rates. Criticism is of course much easier than creation, and while many authors have been critical of the EKC methodology, very little has been offered as a productive alternative.

In this section we present an alternative method to investigate the growth and environment relationship that draws on existing work in macroeconomics on absolute and conditional convergence. Our goal is to develop an explicit link between theory and empirical estimation since this link is largely absent in this literature. A secondary goal is to demonstrate Green Solow's ability to explain cross-country patterns of emissions growth with relatively few variables.

The Green Solow model contains two empirical predictions regarding convergence in emissions. The first is that a group of countries sharing the same parameter values - savings rates, abatement intensities, rates of technological progress etc. - but differing in initial conditions will exhibit convergence in a measure of their emissions. This is true even though each of these countries would typically exhibit a unique income and pollution profile over time. We will derive an estimating equation below to show that under the assumption of identical parameters values across countries, we obtain a prediction of *Absolute Convergence in Emissions* or ACE. Under the ACE hypothesis differences in the pattern of cross-country growth of emissions per person, is fully explained by differences in initial emissions per person. This prediction follows from the familiar forces of diminishing returns plus an