6 Appendix

6.1 Constant costs and rising standards

It proves useful to adopt a specific abatement production function and consider the firm's problem. To that end, let the intensive abatement production function be given by $a(\theta) = [1-\theta]^{\varepsilon}$ where $\varepsilon > 1$. Assume the government imposes a technology restriction requiring emissions per unit of output not exceed $\mu(t)$; that is, $E(t)/Y(t) \leq \mu(t)$. If firm's rent capital at rate r, hire labor at the wage w, and face the technology standard $\mu(t)$, then the firm's problem becomes one of maximizing profits by choice of labor, capital and abatement inputs

$$\begin{aligned} \underset{\{k,l,\theta\}}{Max}\Pi &= Y - wL - rK & (29) \\ s.t. Y &= (1-\theta)F(K, BL) & (30) \\ E &= \Omega[1-\theta]^{\varepsilon}F, & (31) \\ E/Y &\leq \mu & (32) \end{aligned}$$

Since abatement is costly in terms of foregone output firms will only just meet the technological standard. Using this information we can substitute the constraints into the objective to rewrite the firm's problem as simply

$$\underset{\{k,l\}}{Max}\Pi = (\mu/\Omega)^{1/(\epsilon-1)} F(K, BL) - wL - rK$$

which is just a straightforward problem of input choice. The firm's allocation of labor and capital to abatement is determined by the technological standard. Algebra shows

$$(\mu/\Omega) = (1-\theta)^{(\epsilon-1)} \tag{33}$$

which solves for the intensity of abatement implicitly. Note if $\mu = \Omega$, no abatement is necessary and $\theta = 0$, but as standards tigten μ falls and θ must rise. Suppose the technology standard is tightened slowly over time. Differentiating with respect to time we obtain:

$$\frac{\dot{\mu}}{\mu} + g_A = (\epsilon - 1)\frac{d}{dt}\ln(1-\theta)$$

which indicates that the intensity of abatement rises or falls over time as technological progress outstrips or falls behind the steady march of rising technology standards. If the technology standard becomes tighter over time at rate g_A then cost minimizing firms meet the ever tightening standard by allocating the constant fraction θ of their inputs to abatement.

Therefore our fixed θ corresponds to a world where imperfect governments have been raising emission standards slowly over time while firms have been minimizing costs in meeting them. Despite the rising standards, technological progress in abatement has kept pollution control costs roughly constant as a fraction of overall activity.

6.2 Proofs to Propositions

Proposition 1. If growth is sustainable and $k^T > k(0)$, then the growth rate of emissions is at first positive but turns negative in finite time. If growth is sustainable and $k(0) > k^T$, then the growth rate of emissions growth is negative for all t. If growth is unsustainable, then emissions growth declines with time but remains positive for all t.

Proof: From 10 and 5 the growth rate of emissions is declining in k. By definition emissions growth is zero at k^T. Therefore, if $k^T > k(0)$ growth is positive but declines with k; if $k^T < k(0)$ growth is negative and declines with k. When growth is sustainable $k^T < k^*$. The solution for $k(t)$ in 14 shows k^T is reached in finite time from $k(0) < k^T$. If growth is not sustainable, $k^T > k^*$. The solution for $k(t)$ shows it converges to k^* as time goes to infinity. This implies $k^T > k^*$ always, and by definition of k^T emission growth remains positive.

Proposition 2. Proof in the text

Proposition 3. There exists a parametric relationship between emissions E and income per capita y^c that we refer to as an EKC. If $k^* > k^T > k(0)$, then emissions first rise and then fall with income per capita. If $k^* > k(0) > k^T$, then emissions fall monotonically with income per capita.

Proof: We note from the text that

$$\begin{aligned} E(t) &= c_0 \exp[G_E t] \left[\left[k^{*(1-\alpha)}(1-\exp[-\lambda t]) + k(0)^{1-\alpha}\exp[-\lambda t]\right]^{\alpha/(1-\alpha)} \right] \\ y^c(t) &= k(t)^\alpha B(0)[1-\theta]\exp[gt] \end{aligned}$$

We have already shown that for any $k(0) < k^*$, $k(t)$ is increasing in time. Given the properties of exp we can then conclude that $y^c(t) = [1-\theta]k(t)^\alpha B(0)\exp[gt]$ is strictly increasing in time when the conditions of the proposition are met. This allows us to invert and obtain $t = \varphi(y^c)$ where $\varphi' > 0$. Substitute for time in $E(t)$. Now differentiate this parametric function $E(\varphi(y^c))$ with respect to y^c to obtain $E'(\varphi(y^c))\varphi'(y^c)$. Note that if $k^* > k^T > k(0)$ then $E'(\varphi(y^c))$ is positive for $t < T$, zero at $t = T$, and negative for $t > T$. $\varphi'(y^c)$ is always strictly positive and hence emissions at first rise and then fall with income per capita. If $k^* > k(0) > k^T$,then $E'(\varphi(y^c))$ is always negative. This implies $E'(\varphi(y^c))\varphi'(y^c)$ is always negative as required.

6.3 Derivation of Estimating Equation

Start with the capital accumulation equation in 5 and use the following log linearization

$$\frac{\dot{k}}{k} = sk^{\alpha-1}[1-\theta]-[\delta+n+g] \tag{34}$$

$$\cong s[k^*]^{\alpha-1}[1-\theta][1+(\alpha-1)\hat{k}]-[\delta+n+g] \tag{35}$$

$$\hat{k} = \log k(t)-\log k^* \tag{36}$$

Rearrange and use the definition of k^* and λ to obtain the simpler form

$$\frac{\dot{k}}{k} = -(1-\alpha)[n+g+\delta]\hat{k} = -\lambda\hat{k} \tag{37}$$

where we have used the fact that $\dot{k}$ is zero at k^*. Note that $\hat{k} = \log[k(t)/k^*]$ and the left hand side of 37 is just the time derivative of $\log k(t)$. To change this differential equation in $\log k(t)$ to one over $y(t)$ use $y = (1-\theta)k^\alpha$ and then rewrite it out more completely as

$$\frac{d}{dt}[\log y(t)] = -\lambda \log y(t) + \lambda \log y^* \tag{38}$$

This equation is easily solved to find

$$\log y(t) = \log y(0)e^{-\lambda t} + \log y^*[1-e^{-\lambda t}] \tag{39}$$

where $y(0)$ is income per effective worker at $t=0$. Evaluate 39 at T and $T-N$. Note $T-N$ is our initial period and corresponds to $t=0$, since $y(0)$ is period $T-N$ income per effective worker. Doing so we obtain

$$\log y(T) - \log y(T-N) = -\log[y(T-N)/y^*][1-e^{-\lambda N}] \tag{40}$$

Note that income per effective worker is related to income per worker by $y(t) = y^c(t)/B(t)$ where $B(t)$ is the index of labor augmenting technological progress. Making these substitutions leads to

$$\log y^c(T) - \log y^c(T-N) = Ng - \log[y^c(T-N)/B(T-N)y^*][1-e^{-\lambda N}] \tag{41}$$

Divide both sides by N to obtain the average log changes over the period and rearrange slightly to obtain

$$
\begin{aligned}
\frac{\log[y^c(T)/y^c(T-N)]}{N} &= \left[g + \log B(T-N)\frac{[1-e^{-\lambda N}]}{N} + \log[y^*]\frac{[1-e^{-\lambda N}]}{N}\right] \\
&\quad - \log[y^c(T-N)]\frac{[1-e^{-\lambda N}]}{N}
\end{aligned}
$$

which is reported in 22 where the constant b represents the first three terms in brackets.

7 Data

We have obtained our data from several sources. Data on carbon emissions, carbon per capita, carbon per dollar GDP, population size, and investment as a share of GDP was obtained from the World Bank Development Indicators 2002 available on CD-ROM. The data in Figure 7 is drawn from this source. The countries that appear in Figure 7 are: Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Argentina, Australia, Austria, Bahamas, Bahrain, Barbados, Belgium, Belize, Benin, Bolivia, Brazil, Brunei, Bulgaria, Burkina Faso, Cambodia, Cameroon, Canada, Cape Verde, Chad, Chile, China, Colombia, Congo Dem. Rep.,Congo Rep., Costa Rica, Cote d'Ivoire, Cuba, Cyprus, Denmark, Djibouti, Dominica, Dominican Republic, Ecuador, Egypt, Arab Rep.,El Salvador, Equatorial Guinea, Ethiopia, Fiji, Finland, France, French Polynesia, Gabon, Gambia, Ghana, Greece, Greenland, Grenada, Guam, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, China, Hungary, Iceland, India, Indonesia, Iran, Islamic Rep., Iraq, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Korea, Dem. Rep., Korea, Rep., Kuwait, Lao PDR, Lebanon, Liberia, Libya, Luxembourg, Macao, China, Madagascar, Mali, Malta, Mauritania, Mauritius,, Mexico, Mongolia, Morocco, Mozambique, Myanmar, Nepal, Netherlands, New Caledonia, New Zealand, Nicaragua, Niger, Nigeria, Norway, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Puerto Rico, Qatar, Romania, Samoa,Sao Tome and Principe, Saudi Arabia, Senegal,Sierra Leone, Singapore, Solomon Islands, South Africa, Spain,Sri Lanka,,St. Lucia,St. Vincent and the Grenadines, Sudan, Suriname, Sweden, Switzerland, Syrian Arab, Republic, Thailand, Togo, Tonga, Trinidad and Tobago, Tunisia, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Uruguay, Venezuela, RB,Virgin Islands (U.S.).

Data on US emissions of the criteria pollutants graphed in Figures 1 and 2 come from the US E.P.A. The long series of historical data presented in the figures is taken from the EPA's 1998 report National Pollution Emission Trends, available at http://www.epa.gov/ttn/chief/trends/trends.9

Data on European pollution emissions given in Table 3 comes from the monitoring agency for LRTRAP available at http://www.emep.int/.

Data on pollution abatement costs came from the 1996 and 2003 OECD publication Pollution Abatement and Control Expenditures in OECD Countries, Paris: OECD Secretariat. Since this data is difficult to get we have given the exact method of construction and the data used in the table below.

OECD Pollution Abatement Cost Estimates and Sources

Country	θ	Group	Year	OECD Source
Netherlands	1.7	P&P	1985, 1987, 1989-1992	1996
Japan	1.1	P&P	1985-1990	1996
Italy	.9	P&P	1989	1996
Ireland	.6	P&P	1998	2003
Greece	.5	Pub.	1985-1991	1996
France	1.2	P&P	1985-1992	1996
U.S.A.	1.7	P&P	1985-1992	1996
Luxembourg	.4	Pub.	1997	2003
U.K.	1.5	P&P	1990	1996
Switzerland	2.1	P&P	1992	1996
Sweden	1.2	P&P	1991	1996
Spain	.5	Pub.	1987-1991	1996
Portugal	.6	P&P	1988-1991	1996
Norway	1.2	P&P	1990	1996
New Zealand	.9	P&P	1990	1996
Finland	1.4	P&P	1992	1996
Denmark	.6	Pub.	1985-1991	1996
Canada	.9	P&P	1989	1996
Belgium	1.4	P&P	1996-2000	2003
Austria	1.6	P&P	1985,1987,1988, 1990-1991	1996
Australia	.9	P&P	1991	1996
Iceland	.3	Pub.	1985-1992	1996

Notes: P&P refers to both public and private expenditures. OECD source refers to whether the figures come from the 1996 or 2003 OECD study. In constructing these data two rules were followed. First we relied on the 1996 study as it had the longest time series and the time frame fit closer to the middle of our sample period. Second, in some cases data was not available in the 1996 study. In these cases, we then used the 2003 study. This was true for example for Belgium, Luxembourg, and Ireland. Third, we used the most inclusive measure reported. Public and Private is more inclusive than just private, although the definitions of public and private differ across countries. Finally, in some cases we calculated the figures ourselves. New Zealand was given the same ratio of expenditures as Australia, and Luxembourg's ratio was calculated by hand using numbers from the OECD (2003) publication plus data on real GDP in 1997.