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'Quality ladders' and Ricardian trade

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A model of endogenous growth and trade is developed that extends the continuum Ricardian model of Dornbusch et al. (1977) to a dynamic framework, generalizes the 'quality ladders' approach of Grossman and Helpman (1991a, b), and complements the work of Krugman (1987) on dynamic Ricardian economies. In contrast to earlier work the model incorporates heterogeneity across industries in research and production technologies, and in the technological opportunity for innovation. The importance of heterogeneity is demonstrated through a comparative steady-state analysis. Several applications for the model are discussed and many others appear possible given its relatively simple structure.

1. Introduction

In several recent articles Gene Grossman and Elhanan Helpman have developed a series of new models of endogenous growth and trade stressing the role of continual product innovation. In their 'quality ladders' formulation an endless series of innovations leads to improvements in the quality of existing products, while in their 'love of variety' formulation an endless series of innovations leads to an increase in the breadth of the product spectrum.[1] Despite the apparent differences between these models, the authors show the two models share identical reduced forms, and both exhibit properties similar to those of the standard Heckscher–Ohlin model of international trade. The purpose of this paper is to build on the 'quality ladders' approach directly, and construct a dynamic analog of the continuum Ricardian model of Dornbusch et al. (1977).

My approach borrows heavily from Grossman and Helpman's earlier work; consequently, the results presented here demonstrate the ability of the

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[1]The 'quality ladders' formulation is presented in Grossman and Helpman (1991a, b), while the 'love of variety' approach is adopted in Grossman and Helpman (1990, 1991c, d).

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'quality ladders' framework to encompass both the Heckscher–Ohlin and Ricardian models of international trade. Moreover, given the relative simplicity of the continuum Ricardian model, and its proven usefulness in examining many issues, it is hoped that this new dynamic formulation may prove useful in many other contexts as well. A simple comparative steady-state analysis is undertaken to demonstrate the model's basic properties, and other applications of the model are suggested.

In Grossman and Helpman (1991a) all goods enter the consumers' utility function symmetrically, all goods are produced under identical cost conditions, and all research efforts targeted at improving these goods are equally productive. Consequently, the distribution of R&D activity across industries is uniform. This result is, however, at odds with two 'stylized facts' regarding industry characteristics and R&D intensities. These are: (1) R&D intensities differ widely across industries with R&D heavily concentrated in a few sectors of the economy; and (2) the intensity of R&D activities across industries is correlated with both 'demand pull' and 'technology push' factors.[2] In an attempt to reconcile these very basic empirical regularities with a theory of aggregate growth, I present a suitably generalized version of the one-factor 'quality ladders' model.

The model differs from previous work because industries differ in production and research technologies, in the technological opportunity for improvements in technologies, and in the expenditure share allocated to each final good. Introducing this heterogeneity into the 'quality ladders' framework provides us with a model exhibiting both of the empirical regularities cited previously. Apart from heterogeneity, the model is identical to the one-factor model of Grossman and Helpman (1991a). Therefore, while I model innovation as lowering unit production costs rather than raising product quality, this difference is not substantive.

Apart from its empirical role, heterogeneity plays two very important theoretical roles. First, by introducing heterogeneity into the 'quality ladders' framework I obtain a dynamic two-sector version of the continuum Ricardian model of Dornbusch et al.[3] The model, like its static predecessor, is easy to work with and many results can be derived graphically. Second, heterogeneity across the high-tech industries allows for intraindustry resource reallocations. With perfect symmetry across industries one segment of industries cannot expand while another contracts. This simple observation has far-reaching implications. For example, in Grossman and Helpman (1991d) the authors show a subsidy to goods production in the high-tech sector can slow economic growth since an expansion of high-tech manufacturing requires a contraction of R&D production if both make intensive use

[2]Cohen and Levin (1989) and Kamien and Schwartz (1982) discuss the competing 'technology push' and 'demand pull' theories of innovation.

[3]It is also similar to the model developed in Krugman (1990, pp. 165–183).

of a common factor. Similar reasoning holds in Rivera-Batiz and Romer (1991) where symmetric tariffs on home and foreign final goods shift resources into manufacturing and lower growth.

These results necessarily follow because any uniform subsidy or tariff policy must have uniform effects on all industries within the high-tech sector. Heterogeneity within the high-tech sector removes this constraint. The simplest way to demonstrate this feature of the continuum Ricardian model is to consider uniform changes in production and R&D technologies, and note the non-uniform effects. To this end I present a simple comparative steady-state analysis showing that in general uniform changes in technologies often require both intraindustry and interindustry resource reallocations. I suspect that similar results would follow in more complicated policy experiments.[4]

The remainder of the paper proceeds as follows. Section 2 presents the basic assumptions and describes the model's autarky solution. Section 3 then introduces a two-country version with trade in both final goods and R&D results. In section 4 I undertake a comparative steady-state analysis, while section 5 contains suggestions for future research and a brief conclusion. All detailed derivations are relegated to an appendix available upon request from the author.

2. Autarky equilibrium

2.1. Consumers

I assume a single primary factor exists in fixed and inelastic supply. Consumers are endowed with this factor, denoted by L, and share an identical, time separable, and homothetic utility function defined over a continuum of final goods indexed by z. One such representative consumer maximizes the expected discounted value of lifetime utility given by

$$U = \mathrm{E}_0\left[\int_0^{\infty} \mathrm{e}^{-\rho t} \ln u(t)\, \mathrm{d}t\right],$$

where

$$\ln u(t) = \int_0^1 b(z) \ln[x(z,t)]\, \mathrm{d}z. \tag{1}$$

E_0 is the expectation taken at time zero conditional on current information, $x(z,t)$ is the quantity of good z consumed at time t, and ρ is the rate of time

[4]In Taylor (1991a) I show that the effect of a change in intellectual property rights regime depends critically on the cross-country and within-country heterogeneity in research and production technologies, and in the market shares for products.

preference. $b(z)$ is the continuum counterpart to the many-commodity budget share for good z, and satisfies

$$1=\int_0^1 b(z)\,\mathrm{d}z, \quad \mathrm{d}B(z)=b(z)\,\mathrm{d}z, \quad B(1)=1, \quad B(0)=0. \tag{2}$$

Since (1) is additively separable and $u(t)$ is homothetic, two-stage budgeting is appropriate. In the second budgeting stage the consumer maximizes per-period utility subject to a given level of expenditures $\mathrm{E}(t)$. Solving this problem by variational methods yield $x(z,t)=b(z)\mathrm{E}(t)/p(z,t)$ for $z\in[0,1]$. By employing these demand functions in (1), the consumer's first-stage maximization problem is now solved by choosing the pattern of spending $\mathrm{E}(t)$ subject to an intertemporal budget constraint.

Consumers smooth their expenditures over time by investing in the securities offered by firms active in innovation. The return to these shares is uncertain, but their risk is idiosyncratic. Therefore, consumers can eliminate all variance from their portfolios by holding stock in each of the continuum of firms. Consequently, the intertemporal budget constraint takes the familiar form $\mathrm{d}A(t)/\mathrm{d}t=r(t)A(t)+w(t)L-\mathrm{E}(t)$, where $r(t)$ is the certain return on consumers' portfolio. Solving the consumer's first-stage problem by variational methods yields the equation of motion for period expenditures $[\mathrm{dE}(t)/\mathrm{d}t]/\mathrm{E}(t)\equiv\hat{E}=r(t)-\rho$.

2.2. Technology

I assume the economy possesses a continuum of goods indexed by z over the support $[0,1]$. Each good is produced by labor power alone, but with methods reflecting the state of technology currently employed in industry z. When generation j technology is applied in industry z, the unit labor requirements in goods production become

$$a(z,j)=a(z)\phi(j,z), \quad \text{for } j\in(0,1,2,\ldots), \text{ and } z\in[0,1], \tag{3}$$

where $a(z)$ is a pure labor cost component and $\phi(j,z)$ is the component capturing the impact of future innovations. At time $t=0$, generation $\phi(j=0,z)$ technologies are already in place, whereas generations $\phi(j>0,z)$ technologies are yet to be discovered. I assume the set of all future technologies j is countably infinite with innovators worldwide racing to develop the $j+1$st generation if generation j is already in place.

I introduce a strong ordering on technologies by requiring later generations to dominate earlier ones by virtue of their lower labor requirements. The magnitude of this dominance is determined by the inventive step $n(z)$. I assume $n(z)$ is continuous in z, is constant over time, but may vary across industries. For every industry z, the relationship between $n(z)$, $\phi(j,z)$, and $\phi(j+1,z)$ is governed by

$$\phi(j+1,z)\equiv[1-n(z)]\phi(j,z), \quad \text{where } 1>n(z)\geqq 0 \text{ and } j\in(0,1,2,\ldots). \quad (4)$$

If an innovator is successful in discovering the next generation of technology for some industry z', then she obtains exclusive control over the new $\phi(j+1,z')$ technology and adds to the generally available 'know how' in industry z'. I assume new technologies created for industry z' are only useful in z', and 'know how' spillovers are industry specific. In total, these assumptions describe an R&D discovery process where a continuous sequence of patent races occurs in each industry. Every innovation brings with it an industry specific 'know how' spillover that enables subsequent innovators to improve upon the state of the art. These complementary 'know how' inputs accumulate, and hence future generations of technology build on earlier ones.

2.3. *Market structure*

I associate firms with two activities: R&D and production. Firms select from the set of z industries a portfolio of research projects to undertake and allocate research to each. I assume firms obtain a patent of infinite duration for any discoveries they make; therefore, imitation by rivals is excluded by assumption. At any time t, a new innovator with state of the art technology j can *at most* charge consumers a price equal to that offered by firms employing earlier generations of technology. For a given per-unit profit margin of $\pi(j-1,z)$ earned by the patent holder of technology $j-1$, the maximum profit margin earned on the jth technology becomes

$$\pi(j,z)+wa(z)\phi(j,z)=\pi(j-1,z)+wa(z)\phi(j-1,z), \quad j\geqq 1. \quad (5)$$

The patent holder of technology j makes zero profits if the left-hand side of (5) exceeds the right, hence the Bertrand solution becomes[5]

$$\pi^{e}(j-1,z)=0, \quad (6)$$

$$\pi^{e}(j,z)=wa(z)\phi(0,z)n(z)[1-n(z)]^{j-1}, \quad j\geqq 1. \quad (7)$$

The new innovator maximizes profit income by extracting the highest per-

[5]I assume collusion between patent holders on various generations of technology does not occur, and patent licensing to arm's-length competitors is not feasible. For a discussion of the ability of firms to collude in a similar environment, see Segerstrom (1991).

unit profit margin possible subject to the potential competition of previous patent holders. I can now write the schedule of aggregate profits accruing to successful innovators by employing the equilibrium profit rate in (7) to obtain

$$\Pi(j,z,t)=\pi^{e}(j,z)x(z,t)=n(z)b(z)\mathrm{E}(t), \quad z\in[0,1]. \tag{8}$$

Eq. (8) links the flow profits from innovation to the primitives of tastes $[b(z)]$ and technology $[n(z)]$. Perhaps the most notable feature of (8) is the absence of the index j – the rewards to innovation are constant across generations of knowledge!

2.4. Innovation

I assume the R&D discovery technology is Poisson with an arrival rate varying proportionately with R&D expenditures. One unit of research at intensity i in industry z requires $a_{\mathrm{I}}(z)$ units of labor. I denote the particular value for R&D effort i chosen in industry z as $i(z)$. If potential innovators[6] in aggregate undertake R&D at level i targeted at discovering the next generation of technology in industry z', for a time $\mathrm{d}t$, then the instantaneous probability of success in z' is approximately $i(z')\,\mathrm{d}t$. If $V(z)$ is the expected present discounted value of an infinite life patent in industry z, then free entry into research requires that expected benefits equal costs. Hence, $V(z)=wa_{\mathrm{I}}(z)$ when $i(z)>0$.

To fund their R&D investments, firms sell equity shares to consumers. Shares from successful firms pay dividends at rate $\Pi(z)\,\mathrm{d}t$, earn capital gains at rate $[[\mathrm{d}V(z)/\mathrm{d}t]/V(z)]\,\mathrm{d}t$ and suffer a capital loss of $wa_{\mathrm{I}}(z)$ with probability $i(z)$. Hence, the expected rate of return earned on the shares of firms in industry z becomes

$$r(z)=[\Pi(z)+\mathrm{d}V(z)/\mathrm{d}t-wa_{\mathrm{I}}(z)i(z)]/V(z). \tag{9}$$

We can rewrite the expected rate of return schedule by combining (9) and (8), and recalling that with each project's risk idiosyncratic, the expected rate of return $r(z)$ must equal the risk-free rate on the portfolio r.[7] Choosing

[6]Industry leaders to not undertake R&D. See the discussion in Grossman and Helpman (1991a, p. 47) on this point.

[7]Introduce a hypothetical risk-neutral arbitrager who pays consumers r and earns an expected rate $r(z)$ from all R&D projects. The arbitrager's portfolio consists of a continuum of projects with expected excess rates of return $X(z)=r(z)-r$. Competition amongst arbitragers raises r to $r(z)$, hence $X(z)$ has a zero mean. If the variance of $X(z)$ is not too large, then Theorem 4.3 in Ross (1988, p. 350) is applicable and a portfolio of these z stocks has a certain return of r with probability one.

$w=1$ for all t implies $\mathrm{d}V(z)/\mathrm{d}t=0$. Hence for all z industries active in R&D, $r(z)$ becomes[8]

$$r=r(z)=n(z)b(z)\mathrm{E}(t)/a_1(z)-i(z), \quad z\in[0,1]. \tag{10}$$

By combining (10) with the consumer's equilibrium condition we obtain a differential equation linking the flow rate of aggregate spending to the flow rate of profits from innovation in each z industry:

$$\hat{E}=n(z)b(z)\mathrm{E}(t)/a_1(z)-i(z)-\rho, \quad z\in[0,1]. \tag{11}$$

With the consumer and capital market equilibrium conditions met in (11), I only need to ensure the labor market clears to close the model. Employment in manufacturing is independent of the generation of technology in use, and recalling $w=1$ the labor market equilibrium can be written as

$$L=\int_0^1 b(z)[1-n(z)]\mathrm{E}(t)\,\mathrm{d}z+\int_0^1 a_1(z)i(z)\,\mathrm{d}z. \tag{12}$$

Eqs. (11) and (12) are two equations in $\mathrm{E}(t)$ and the $i(z)$ profile. Multiply eq. (11) by $a_1(z)$ and integrate over $[0,1]$. Then substitute the result into (12), and rearrange to obtain

$$\hat{E}=[\mathrm{E}(t)-L]/VP-\rho, \quad \text{where } VP\equiv\int_0^1 a_1(z)\,\mathrm{d}z. \tag{13}$$

Eqs. (12) and (13) are analogous to eqs. (10) and (11) in Grossman and Helpman (1991a, p. 49). The systems of equations are identical if $a_1(z)=a_1$, $[1-n(z)]=[1-n]=1/\lambda$, and $b(z)=1$. In terms of their notation, a_1 is the common cost of conducting one unit of R&D in any industry at intensity i, λ is the common step up the quality ladder made with each subsequent innovation, and $b(z)=1$ since all products receive an equal budget share. The solution to (12) and (13) also follows that of Grossman and Helpman's formulation with immediate convergence to a steady state characterized by

$$E=L+\rho VP, \tag{14}$$

$$i(z)=n(z)b(z)[L+\rho VP]/a_1(z)-\rho\geqq 0, \quad z\in[0,1]. \tag{15}$$

The equilibrium growth path is characterized by a constant expenditure level, an unchanging division of labor between manufacturing and R&D

[8]The R&D intensity, $i(z)$, need not be positive over all ranges of z. If $i(z)$ were zero over some segments, then $r(z)=0$ over these segments as well. All integrals involving $i(z)$ would need to be written as sums of integrals over the segments where $i(z)$ is strictly positive. To abstract from these difficulties, I assume $i(z')$ is positive for all z'.

activities, and continual process improvement in all industries where $i(z)>0$. Steady-state expenditures are given by the sum of factor plus profit income, while the steady-state schedule of R&D intensities reflects both 'demand pull' and 'technology push' factors. With $w=1$, L represents factor income, while ρ is the steady-state return on consumer's portfolio of assets. The total value of this portfolio, VP, is given by the integral of $V(z)=a_1(z)$ over the segment [0, 1]. Moreover, solving for the expected growth rate in per-period utility shows[9]

$$\hat{u}(t)=-\int_0^1 \{b(z)\ln[1-n(z)]i(z)\}\,\mathrm{d}z>0. \qquad (16)$$

Since $\ln[1-n(z)]<0$, the economy exhibits perpetual growth. Successive generations of innovators displace the existing patent holders by limit pricing. The benefits of this tumultuous competition befall consumers who observe steadily declining prices for some or all goods. As a result, continual growth in per-capita utility ensues.

In contrast to the 'quality ladders' formulation the distribution of R&D intensities is not uniform across industries. If we associate the technological opportunity present in any industry with the size of the inventive step achieved by any successful innovation, then holding constant market size, eq. (15) predicts a positive relationship between R&D expenditures and technological opportunity. Similarly, holding constant both $n(z)$ and $a_1(z)$ we find R&D intensities respond positively to 'demand pull' factors as captured by $b(z)$. Examining (15) also shows that $i(z)$ need not be positive even though positive flow profits would follow from innovation in industry z. If the return to innovation is not large relative to ρ, then no innovation occurs despite the fact that $n(z)>0$. Consequently, growth can vary across sectors with technologically stagnant sectors ($n(z)\cong 0$) experiencing no growth at all, and technologically dynamic sectors experiencing perhaps double-digit growth.

3. International trade

With these preliminary results in hand, the extension to a trading environment is relatively straightforward. Consider a two-country world where financial capital is internationally mobile and R&D can be undertaken in either country. Label the z industries in order of declining home country comparative advantage in goods production. Then letting an asterisk denote a foreign variable we can construct the schedules of relative labor productivities in goods and R&D production at time zero as follows:

[9]The derivation follows that given in Grossman and Helpman (1991a).

$$A(z) \equiv a^*(z)\phi(0,z)/a(z)\phi(0,z), \quad z \in [0,1], \tag{17}$$

$$RD(z) = a_1^*(z)/a_1(z), \quad z \in [0,1], \tag{18}$$

where $A(z)$ and $RD(z)$ are continuous in z by assumption. $A(z)$ is declining in z by construction, whereas $RD(z)$ may, in general, be a non-monotonic function of z.

I retain the traditional Ricardian assumption of immobile technologies, and require firms target their R&D expenditures towards improving any specific $\phi(j,z)$ component. I assume that improvements to any $\phi(j,z')$ can be implemented on either home or foreign production technologies for z'. While it makes little difference to the results presented here, I also assume that the knowledge spillovers created by successful innovation in industry z' aid subsequent innovators in either country.

Before proceeding to a discussion of the trading equilibrium it is necessary to introduce further assumptions on the $RD(z)$ schedule. While $RD(z)$ may exhibit alternating positively and negatively sloped segments or may be horizontal throughout, I adopt the following three simplifying assumptions to prevent the analysis in section 4 from becoming taxonomic.[10] These are: (1) $RD(z)$ is monotonic in z; (2) $RD(z)$ is falling in z; and (3) $A(z') > RD(z')$ for all $z' \in [0,1]$.

Assuming $RD(z)$ is monotonic in z eliminates the possibility that any given country is the least cost R&D producer over segments of $[0,1]$ that are not contiguous. Assuming $RD'(z) < 0$ amounts to assuming a country's pattern of comparative advantage in goods and R&D production is positively correlated, although not perfectly so. To understand the utility of assumption (3) consider any given $\omega \equiv w/w^*$ and define z^a and z^b as $\omega \equiv A(z^a)$ and $\omega \equiv RD(z^b)$. Then if $z^a > z^b$ I define the home country as having a relative advantage in goods versus R&D production at ω, while the foreign country has a relative advantage in R&D versus goods production at ω. Assuming $A(z') > RD(z')$ for all $z' \in [0,1]$ ensures these definitions *hold for all* ω, and just neatly divides the world into a home country with a relative advantage in goods production and a foreign country with a relative advantage in R&D.

With these assumptions in mind recall that for any given ω, the $A(z)$ schedule at time zero sets the competitive margin in goods production. Retaining the normalization $w=1$ for all t, $\tilde{z}$ is defined by $\omega = 1/w^* = A(\tilde{z}) = a^*(\tilde{z})/a(\tilde{z})$. Similarly, the competitive margin in R&D production $\bar{z}$ is defined by $\omega = 1/w^* = RD(\bar{z}) = a_1^*(\bar{z})/a_1(\bar{z})$. Since both $RD(z)$ and $A(z)$ are monotonically declining in z, any given ω divides the world's available technologies into two sets: the set of *front line technologies* and the set of

[10]While the specific results derived here naturally follow from these assumptions, nothing in the analysis in general relies on them. Other cases are relatively straightforward to examine and are left to interested readers.

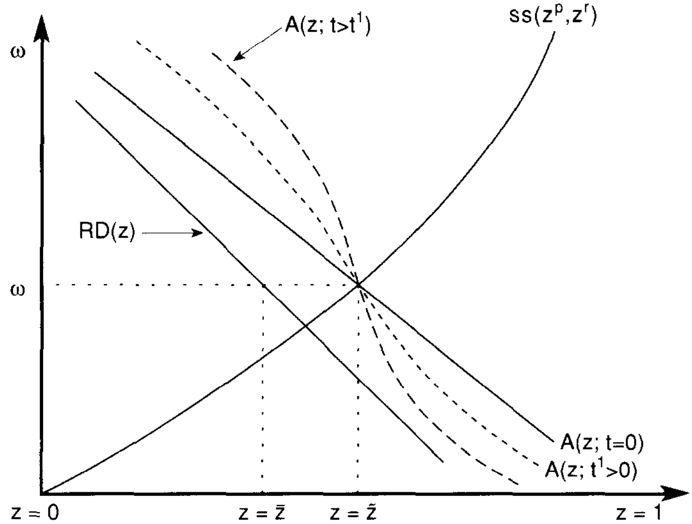

Fig. 1.

backward technologies. Front line technologies are those that are minimum cost given the prevailing wage rates.

In order to analyze trading situations I initially take ω as fixed to determine the division of world production and R&D across countries. Given this assumed division of production and research I derive two dynamic equations governing worldwide spending. Solving these equations subject to budget constraints shows world expenditure must be constant over time and balance of payments requires $\omega = SS(z^{p}, z^{r})$. $\omega = SS(z^{p}, z^{r})$ gives the terms of trade ω needed to maintain balance of payments equilibrium when the home country conducts goods production over $z \in [0, z^{p}]$ and undertakes R&D in industries $z \in [0, z^{r}]$.

For a fixed $\bar{z} = z^{r}$, the $SS(z^{p}, \bar{z} = z^{r})$ schedule operates much like the balance of trade schedule in Dornbusch et al. (1977). Combining $SS(z^{p}, \bar{z} = z^{r})$ with the initial $A(z)$ schedule yields a 'candidate' for the equilibrium terms of trade $\omega = A(\tilde{z}) = SS(\tilde{z}, \bar{z} = z^{r})$. If at this 'candidate' ω it is also true that $\omega = RD(z^{r})$, then ω, $\bar{z}$, and $\tilde{z}$ are indeed the steady-state equilibrium values. Alternatively, if at this 'candidate' ω we have $\omega > RD(z^{r})$, then a greater segment of R&D should be conducted abroad. Hence $\bar{z}$ must be less than z^{r}. Once we have found a ω, $\bar{z}$, and $\tilde{z}$ consistent with all three schedules we have completely characterized the steady-state division of world production and R&D. The steady state is realized immediately, and the dynamic evolution of the world economy evolves as shown in fig. 1.

Fig. 1 presents a graphical treatment of the evolving world economy. At time zero the $A(z)$, $RD(z)$ and $SS(z^{\mathrm{p}}, z^{\mathrm{r}})$ schedules together determine the competitive margins $\tilde{z}$ and $\bar{z}$, and the initial terms of trade ω. Innovators undertake improvements to the $\phi(j,z)$ components, and incentives lead them to implement these on front line production technologies. Hence, successful innovation lowers all $a(z')\phi(j,z)$ for $z'<\tilde{z}$ and all $a^*(z'')\phi(j,z)$ for $z''>\tilde{z}$. These actions gradually deform the $A(z)$ schedule. The changes in $A(z)$ brought about by research successes drive worldwide growth, but leave the competitive margins unchanged, the balance of payments in equilibrium, and only amplify the original differences in unit labor requirements.

To derive the world equilibrium depicted in fig. 1 I proceed as follows. At $t=0$, the technologies $\phi(j=0,z)$ are already in place. At some time $t>0$, assume $\phi(1,z')$ is discovered by a foreign firm and let $a^*(z')$ be the corresponding front line technology. The foreign patent holder on $\phi(1,z')$ can implement this improvement on the foreign technology $a^*(z')$ and earn a profit margin of $\pi^{\mathrm{e}}(1,z)=w^*\phi(0,z)a^*(z)n(z)$ over the entire world demand for product z'. Flow profits from this strategy are given by $\Pi(1,z',t)=b(z')n(z')[E^*+E]$. Conversely, the successful foreign innovator could 'go multinational' and carry the innovation abroad to a wholly owned subsidiary. This subsidiary would then pay the foreign firm a royalty of $\pi(j,z)$ per unit of output for the exclusive right to implement the improvement on the domestic technology $a(z')$. This strategy earns $\Psi(1,z',t)=[1-[1-n(z')]\omega/A(z')]b(z')[E^*+E]$. Rearranging shows $\Pi(1,z',t)>\Psi(1,z',t)$. Therefore, innovations are always implemented on front line production technologies. For expositional purposes when innovation and implementation occur in different countries I refer to the resulting intrafirm transactions as imports and exports of R&D.

Next, consider the innovation process itself. Suppose simultaneous targeting of any $\phi(j,z')$ took place in both countries. Then the potential flow profits from innovation are identical for both innovators, but the costs of conducting R&D are lower in the country with the corresponding front line R&D technology. With a linear research technology, only innovators in the country where the R&D technology is front line can raise the funds necessary for innovation.[11] With these results in hand we can now write the rate of return schedule $r(z)$ as

$$r(z)=n(z)b(z)[E+E^*]/a_{\mathrm{I}}(z)-i(z), \quad z\in[0,\bar{z}], \tag{19}$$

$$r(z)=n(z)b(z)[E+E^*]/w^*a_{\mathrm{I}}^*(z)-i^*(z), \quad z\in[\bar{z},1]. \tag{20}$$

Consumers' equilibrium condition requires $\hat{E}=r-\rho$ and $\hat{E}^*=r^*-\rho$. Perfect international capital mobility implies $r=r^*$, hence $\hat{E}=\hat{E}^*$. Therefore,

[11]See Grossman and Helpman (1991b).

combining (19) and (20) with $\hat{E}=r-\rho$ and $\hat{E}^*=r^*-\rho$ and solving as in the autarky model shows[12]

$$[E+E^*]=w^*L^*+L+\rho VP, \tag{21}$$

$$VP=\int_0^{\bar{z}} a_I(z)\,dz+\int_{\bar{z}}^{1} w^*a_I^*(z)\,dz\equiv A_I+w^*A_I^*. \tag{22}$$

The steady state is characterized by $[\hat{E}+\hat{E}^*]=0$ and since $\hat{E}=\hat{E}^*$, this implies $r=\rho$. E and E^* are given by $E=L+\lambda VP$ and $E^*=w^*L^*+[1-\lambda]VP$. Expenditures at home and abroad are given by their respective factor incomes plus their shares of the worldwide returns from innovation. λ is home consumers' share of worldwide assets VP and $[1-\lambda]$ is foreign consumers' share. VP is, in turn, just equal to the aggregate value of innovating firms at home and abroad measured over their respective regions of $[0,1]$.

With $\hat{E}=0$, λVP must be constant; hence $[1-\lambda]VP$ must be constant as well. If both $[1-\lambda]VP$ and λVP are constant then balance of payments requires the current account balance since both countries' net foreign asset positions are constant over time. Therefore, balance of payments requires

$$\left[\int_0^{\bar{z}} b(z)E^*\,dz-\int_{\bar{z}}^{1} b(z)E\,dz\right]$$

$$+\left[\lambda\rho w^*A_I^*-[1-\lambda]\rho A_I-\int_{\bar{z}}^{\tilde{z}} n(z)b(z)[E+E^*]\,dz\right]=0. \tag{23}$$

The first bracketed term in (23) represents the home country's trade balance in goods. The second bracketed term represents the home country's balance in services. A share $B(\tilde{z})$ of foreign expenditures goes to home exports, while $[1-B(\tilde{z})]$ of home expenditures goes to imports. Any deficit in the goods trade balance must be offset by a surplus in the services account. A positive services account requires home consumers' share of the dividends from foreign R&D producers, $\lambda\rho w^*A_I^*$ exceed the sum of dividend payments made by home R&D firms to foreign consumers, $[1-\lambda]\rho A_I$, plus the royalty payments made by home subsidiaries to their foreign parents over $[\bar{z},\tilde{z}]$.

If we substitute for E and E^* and rearrange (23) we obtain the balance of payments schedule $SS(z^p,z^r)$ depicted in fig. 1. Formally $\omega=SS(z^p,z^r)$ is defined by

[12] An appendix containing the intermediate calculations is available upon request from the author.

$$\omega=[[L^*+\rho A_1^*]/[L+\rho A_1]]\left[B(\tilde{z})-\int_{\bar{z}}^{\tilde{z}} n(z)b(z)\,\mathrm{d}z\right]\Big/$$

$$\times\left[1-B(\tilde{z})+\int_{\bar{z}}^{\tilde{z}} n(z)b(z)\,\mathrm{d}z\right]. \tag{24}$$

$SS(z^{\mathrm{p}},z^{\mathrm{r}})$ is upward sloping as a function of $\tilde{z}$ since an increase in the range of commodities produced at home raises exports and must be met with a corresponding increase in relative wages, and hence imports, to maintain balance of payments equilibrium. Alternatively an increase in $\bar{z}$ shifts the entire $SS(z^{\mathrm{p}},z^{\mathrm{r}})$ schedule upwards since an increase in the range of R&D industries active at home raises the demand for home labor for any given $\tilde{z}$. It is apparent from (24) that $SS(z^{\mathrm{p}},z^{\mathrm{r}})$ is rising in $z^{\mathrm{p}}=\tilde{z}$. To confirm that it shifts upwards with an increase in $\bar{z}=z^{\mathrm{r}}$, differentiate to obtain

$$\partial SS(z^{\mathrm{p}},z^{\mathrm{r}})/\partial(\bar{z}=z^{\mathrm{r}})=\omega\rho i(\bar{z})a_1(\bar{z})[L+\rho A_1]\left[1-B(\tilde{z})+\int_{\bar{z}}^{\tilde{z}} n(z)b(z)\,\mathrm{d}z\right]>0. \tag{25}$$

It is also apparent from (24) that the $SS(z^{\mathrm{p}},z^{\mathrm{r}})$ schedule is independent of the distribution of asset ownership across countries, and hence the value of λ plays no role in the model's positive properties. This is just another example of the Transfer Problem. With tastes identical and homothetic, a change in the distribution of assets in the steady state leaves world expenditure unchanged, and the resulting world equilibrium unaltered.

Combining the $SS(z^{\mathrm{p}},z^{\mathrm{r}})$ schedule in (24) with the $A(z)$ and $RD(z)$ schedules determines the equilibrium terms of trade ω, and sets in motion the dynamic evolution of the world economy as depicted in fig. 1. Collecting these results we have:

Proposition 1. If $RD'(z)<0$ and $A(z')>RD(z')$ for all z', then in the steady-state equilibrium the home country produces and exports the range of goods $z\in[0,\tilde{z}]$ and conducts R&D in industries $z\in[0,\bar{z}]$. Moreover, $\tilde{z}>\bar{z}$, hence the home country imports R&D results.

Corollary 1.1. If $RD'(z)<0$ and $A(z')=RD(z')$ for all z', then $\tilde{z}=\bar{z}$, trade in R&D results is zero, but trade in goods still remains.

The equilibrium depicted in fig. 1 shares many common features with both the static continuum model and the dynamic learning-by-doing variant introduced in Krugman (1987). The pattern of trade is determined by comparative advantage across industries, captured by $A(z)$ and $RD(z)$, together with relative country size and other demand considerations reflected in the $SS(z^{\mathrm{p}}, z^{\mathrm{r}})$ schedule. In contrast to the work of Grossman and Helpman (1991a), factor price equalization is not a generic property of the equilibrium. A country's terms of trade will rarely equal 1, and specialization rather than diversification is a key feature of the model. Moreover, in contrast to the equilibria described in Grossman and Helpman (1991b) relative wages are never independent of relative country size, and innovation occurs in both countries despite (perhaps large) differences in absolute advantage across countries.

4. Comparative steady-state analysis

In this section I examine the comparative static properties of the steady-state equilibria presented in fig. 1.[13] To facilitate this analysis I introduce five parameters that shift the $A(z)$, $RD(z)$ and $n(z)$ schedules uniformly. I replace $A(z)$ by $A(z)[\alpha^*/\alpha]$, $RD(z)$ by $RD(z)[\delta^*/\delta]$, $n(z)$ by $vn(z)$, and evaluate all derivatives with parameters set to unity.

Individual changes in the parameters $\{\alpha, \alpha^*, \delta, \delta^*\}$ have three effects. Take for example $\mathrm{d}\alpha > 0$. An increase in α reduces the home country's comparative advantage in goods production, it reduces the home country's absolute advantage in goods production (if any), and it reduces the home country's *relative* advantage in goods versus R&D. Equiproportionate increases in α and α^*, or δ and δ^* leave a country's comparative and relative advantage unaffected, but increase the existing absolute differences in goods or R&D technologies. Finally, an increase in v raises the flow profits expected from innovation at every instant of time. We can write the three equilibrium relationships governing fig. 1 more generally as:

$$\omega \equiv SS(\tilde{z}, \bar{z}; L/L^*, \delta, \delta^*, v), \quad S_1 > 0, S_2 > 0, S_3 < 0, S_4 < 0, S_5 > 0, S_6 < 0, \tag{26}$$

$$\omega \equiv A(\tilde{z}; \alpha, \alpha^*), \quad A_1 < 0, A_2 < 0, A_3 > 0, \tag{27}$$

$$\omega \equiv RD(\bar{z}; \delta, \delta^*), \quad R_1 < 0, R_2 < 0, R_3 > 0. \tag{28}$$

First, consider the role played by the inventive step $n(z)$.[14] A uniform change in $n(z)$ does not affect either country's comparative, absolute, or relative advantage. Nevertheless, the economy's response to $\mathrm{d}v > 0$ depends

[13] Many of the qualitative results presented in this section can be derived graphically using fig. 1.

[14] Comparative statics on country size show $\mathrm{d}\omega/\mathrm{d}L = [S_3 A_1 R_1]/\Delta < 0$, $\mathrm{d}\tilde{z}/\mathrm{d}L = [S_3 R_1]/\Delta > 0$, and $\mathrm{d}\bar{z}/\mathrm{d}L = [S_3 A_1]/\Delta > 0$.

on the existing pattern of trade. To demonstrate this, totally differentiate (26)–(28) to show

$$d\omega/dv=[S_6A_1R_1]/\Delta<0, \quad \text{where } \Delta\equiv R_1[A_1-S_1]-A_1S_2>0, \tag{29}$$

$$d\tilde{z}/dv=[S_6R_1]/\Delta>0, \quad \text{and} \quad d\bar{z}/dv=[S_6A_1]/\Delta>0. \tag{30}$$

Since the home country pays royalties of $n(z)b(z)[E+E^*]$ per unit time in industries $z\in[\tilde{z},\bar{z}]$, an increase in $n(z)$ raises these payments and creates an incipient deficit in the balance of payments. Balance of payments is maintained, however, through two adjustments. First, the home country raises its goods trade balance by increasing the range of goods produced at home, $d\tilde{z}/dv>0$. Second, the home country simultaneously reduces its reliance on imported R&D by conducting more itself, $d\bar{z}/dv>0$.

From (30) it is apparent that the home country is successful in reducing its reliance on foreign-made innovations only if $R_1/A_1<1$. If $R_1/A_1>1$ the measure of goods industries importing results from abroad rises. Not surprisingly, if, at the margin, the home country's comparative advantage in R&D falls rapidly, then the segment of R&D-producing firms active at home rises only minimally in response to the fall in relative wages. As a result, the segment of industries importing R&D results from abroad rises.

These results change dramatically if we eliminate some of the model's heterogeneity. We can remove part of the heterogeneity by eliminating the home country's relative advantage in goods versus R&D by setting $A(z')=RD(z')$ for all $z'\in[0,1]$. Then $\tilde{z}=\bar{z}$, $S_6=0$ and $d\omega/dv=d\tilde{z}/dv=d\bar{z}/dv=0$. If $\tilde{z}=\bar{z}$, then trade in R&D results is zero, and the valuation effect of $dv>0$ is absent. Finally, if we were to go further and eliminate the across country heterogeneity by assuming $A_1=0$, then $\omega=1$ and the patterns of trade in both goods and R&D are indeterminate.

Next, consider the role played by α and α^*. Denote an equiproportionate change in α and α^* by $d\alpha\alpha^*$, then totally differentiate (26)–(28) to obtain

$$d\omega/d\alpha=-A_2S_1R_1/\Delta<0 \quad \text{and} \quad d\omega/d\alpha\alpha^*=0, \tag{31}$$

$$d\tilde{z}/d\alpha=A_2[S_2-R_1]/\Delta<0 \quad \text{and} \quad d\tilde{z}/d\alpha\alpha^*=0, \tag{32}$$

$$d\bar{z}/d\alpha=-A_2S_1/\Delta>0 \quad \text{and} \quad d\bar{z}/d\alpha\alpha^*=0. \tag{33}$$

A uniform increase in the home country's unit production costs reduces its relative wage and reduces the segment of goods-producing industries active at home. With $d\alpha>0$ the home country's relative advantage in goods versus R&D falls and the segment of home industries active in R&D rises. In contrast, the home country's comparative or relative advantage pattern is left unaffected by equiproportionate changes in home and foreign production costs. As expected this removes the need to either relocate production across countries or reallocate labor across sectors.

By employing the same procedures we can examine the role played by δ and δ^*. Totally differentiate (26)–(28) to obtain

$$d\omega/d\delta = -A_2[S_2R_2 - S_4R_1]/\Delta < 0, \tag{34}$$

$$d\tilde{z}/d\delta = -[S_2R_2 - S_4R_1]/\Delta > 0, \tag{35}$$

$$d\bar{z}/d\delta = [R_2S_1 + A_1[\omega L]/[L + A_1]]/\Delta < 0. \tag{36}$$

A uniform increase in R&D costs reduces the home country's relative wage and simultaneously reduces the measure of R&D-producing industries active at home. The resulting fall in home relative wages in turn raises the segment of home industries active in production. Again we find resources flowing into the sector whose relative advantage has been strengthened. The similarity with uniform changes in production technologies ends however once we consider equiproportionate changes in R&D technologies. Denoting an equiproportionate change by $d\delta\delta^*$ we obtain

$$d\omega/d\delta\delta^* = A_1R_1[S_4 + S_5]/\Delta >< 0? \tag{37}$$

$$d\tilde{z}/d\delta\delta^* = R_1[S_4 + S_5]/\Delta >< 0? \tag{38}$$

$$d\bar{z}/d\delta\delta^* = A_1[S_4 + S_5]/\Delta >< 0? \tag{39}$$

The ambiguity evident in (37)–(39) arises because R&D technologies play two roles. In the ratio form $[a_I^*(z)/a_I(z)]$ R&D technologies determine the competitive margin $\tilde{z}$ and the division of R&D across countries. In addition to this role R&D technologies determine, via the free entry process into the R&D, the value of a patent in any industry $V(z)$. Recall $V(z)$ must equal either $w^*a_I^*(z)$ or $a_I(z)$. While equiproportionate changes in δ and δ^* leave the ratio $[a_I^*(z)$ or $a_I(z)]$ unaffected, they raise the steady-state value of patent ownership $V(z)$. The value of holding a patent rises because the productivity of efforts aimed at displacing an incumbent falls. This increase in the value of patents in turn raises world expenditure levels. To investigate the cause of this ambiguity further rearrange (37) to obtain

$$d\omega/d\delta\delta^* = A_1R_1\gamma\left[\left[B(\tilde{z}) - \int_{\bar{z}}^{\tilde{z}} n(z)b(z)\,dz\right]\rho[w^*A_I^* + A_I] - \rho A_I\right]\Big/\Delta >< 0? \tag{40}$$

where γ is a positive constant.[15] $\rho[w^*A_I^* + A_I]$ represents the increase in the

[15] $S_4 = -\omega[\rho A_I]/[L + \rho A_I]$ and $S_5 = \omega[\rho A_I^*]/[L + \rho A_I^*]$. Employ the $SS(z^P, z^r)$ schedule to eliminate $[L + \rho A_I^*]/[L + \rho A_I]$, and rearrange to obtain (40).

flow of returns from the world portfolio of assets $\partial VP/\partial\delta\delta^* > 0$. Equivalently it is equal to $\partial(E+E^*)/\partial\delta\delta^* > 0$. A fraction $B(\tilde{z})$ of this increase falls on home produced goods, but $n(z)b(z)[\partial(E+E^*)/\partial\delta\delta^*]$ for $z \in [\bar{z}, \tilde{z}]$ reverts to foreign hands via royalty payments. Hence, the first term in large brackets represents the increase in world spending captured by the home country. The remaining term, ρA_1, represents that portion of the increase in world income created within the home country. Summing the terms we find that if spending in the home country rises by more than income, an incipient balance of payments surplus is created and ω must rise. Reductions in both $\tilde{z}$ and $\bar{z}$ follow the rise in ω.

In general it is not possible to sign (37)–(39) without further assumptions on the relationship between A_1^* and A_1, and on the distributions governing $b(z)$ and $n(z)$. If we eliminate some of the model's heterogeneity by assuming $a_1^*(z) = a_1(z) = a_1$, then $R_1 = 0$ and ω must be unity in any equilibrium with both countries conducting R&D. Hence $d\omega/d\delta\delta^* = 0$ and $d\tilde{z}/d\delta\delta^* = 0$. If we simplify further by setting $n(z) = n$ and $b(z) = 1$ we obtain

$$d\bar{z}/d\delta\delta^* = A_1\gamma\rho a_1[[1-n][\tilde{z}-\bar{z}]]/\Delta < 0. \tag{41}$$

Since the home country is a net importer of R&D ($[\tilde{z}-\bar{z}] > 0$) the rise in world expenditure falls heavily on its own goods. The incipient balance of payments surplus thereby created cannot be alleviated by a rise in home relative wages; instead, the home country sheds R&D firms. With less R&D conducted at home, royalty payments to foreigners rise and the incipient surplus is eliminated.

In total these comparative steady-state exercises illustrate the tractability of the Ricardian approach, the ease with which determinate results can be obtained, and the importance of comparative, absolute and relative advantage. They also illustrate that even when the model provides ambiguous results, much can be learnt from imposing specific functional forms and proceeding. Lastly, the results demonstrate how heterogeneity guides both intraindustry and interindustry resource reallocations. Clearly further work along these lines could examine how R&D in total and growth rates respond to changes in comparative, absolute or relative advantage. Nevertheless, by concentrating on trade patterns and relative wage rates the reader is given a clear exposition of the model's basic properties, and is immediately drawn to the model's close connection with its static counterpart.

5. Conclusions and suggestions for future research

This paper presented a simple extension of the 'quality ladders' framework to construct a Ricardian model of trade and growth. By introducing this Ricardian version it is now apparent that the 'quality ladders' framework can encompass both the Heckscher–Ohlin and Ricardian models within one general construct. In contrast to earlier work, heterogeneity at the industry level plays a key role in determining the allocation of resources across industries and the pattern of trade across countries. Within an autarkic economy, heterogeneity requires that R&D intensities vary across sectors of the economy with interindustry variations reflecting both 'demand pull' and 'technology push' factors. Within a trading context heterogeneity leads to specialization and a dynamic reformulation of the continuum Ricardian model of Dornbusch et al. (1977).

Given the relative simplicity of the model and the past fruitfulness of the continuum Ricardian approach, this new dynamic formulation may be a useful construct in which to examine other issues. For example, it is relatively straightforward to allow innovators to carry either production or R&D technologies abroad and examine the resulting technology transfer equilibrium. The model's Ricardian structure also allows for a simple examination of the gains from trade in dynamic settings, and it may be amended to examine trading equilibria with imitation and product cycles.[16] Finally, we can investigate the impact of trade and industrial policies on the model's strong predictions for the pattern of trade between countries.

[16]Taylor (1991b) examines the gains from trade within the model. A referee has suggested the product cycle extension.

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