Appendix

Proof of concavity of real income in z

Define $$R(h,z)=\max_{\{x,y\}}\{q(x,y)\ \ s.t.\ \ (x,y)\in\Theta(h,z)\}. \qquad (A1)$$

This is equivalent to our definition of real income in (5) when the economy is closed and p is endogenous since the competitive equilibrium solves the above optimization problem. Let $(h^{\lambda},z^{\lambda}) = \lambda(h^{o},z^{o}) + (1-\lambda)(h^{1},z^{1})$; and let (x^{o},y^{o}) solve (A1) when the endowment is (h^{o},z^{o}), Similarly let (x^{1},y^{1}) solve (A1) for (h^{1},z^{1}), and let $(x^{\lambda},y^{\lambda})$ solve (A1) for $(h^{\lambda},z^{\lambda})$. Then:

$R(h^{\lambda},z^{\lambda}) = q(x^{\lambda},y^{\lambda})\geq q[\lambda(x^{o},y^{o})+ (1-\lambda)(x^{1},y^{1})] \geq \lambda q(x^{o},y^{o}) + (1-\lambda)q(x^{1},y^{1}) = \lambda R(h^{o},z^{o}) + (1-\lambda)R(h^{1},z^{1})$. The first inequality follows since by the convexity of Θ, $\lambda(x^{o},y^{o})+ (1-\lambda)(x^{1},y^{1})$ is feasible but not optimal for $(h^{\lambda},z^{\lambda})$. The second inequality follows since q is concave.

Proof of Proposition 1

Differentiating (9) with respect to Z_{-k} yields, after some rearrangement, an expression for the effect of an increase in rest-of world emissions:

$$1 > -\frac{dz^{k}}{dZ_{-k}} = \frac{MD_z}{MD_z + MD_R R_z - R_{zz} - R_{zp}dp/dz^{k}} > 0.$$

But $MD_R R_z > 0$ from the text, and $-R_{zz} - R_{zp}(dp/dz^{k}) > 0$ by the concavity of R in z (taking into account the endogenous price response) as shown above. This yields the result.

Proof of Proposition 2

Define $T(Z_{-k},p,u)$ as the minimum transfer T needed to implement utility level u:

$$T(Z_{-k},p,u)=\min_{z,T}\left\{T:\ \ u\left(\frac{G(p,h,z)+T}{\Phi(p)},z+Z_{-k}\right)=u\right\}$$

Let $z^c(Z_{-k},p,u)$ be the compensated emissions supply that solves this problem. Then:

$$z\big(Z_{-k}, p, T(Z_{-k}, p, u)\big) = z^c\big(Z_{-k}, p, u\big) \tag{A2}$$

where $z(Z_{-k}, p, T)$ is defined in the text above eq. (10). Differentiating (A2) with respect to p and rearranging yields $z_p = z_p^c - z_T T_p$ But $T_p = m$ from (8) in the text. Hence $z_p = z_p^c - z_T m$.

Substituting this into (10) yields (11).

Stability and Proposition 3. To illustrate the strategic complementarity is not inconsistent with stability, we consider for simplicity the special case where $U_{RZ} = U_{ZZ} = 0$. Using Zhang and Zhang (1996), a sufficient condition for stability is:

$$|U_{RR}(R_z)^2 + U_R R_{zz}| \; > \; |U_{RR} R_p R_z + U_R R_{zp}| \, |\sum_{j \neq k} \partial p / \partial z^j|,$$

where unless otherwise indicated, variables correspond to those for country k. Noting that $R_{zz} = 0$ for our small open economy, and rearranging, the condition is equivalent to:

$$\varepsilon_{MD,R} R_z \; > \; |m\varepsilon_{MD,R} / \Phi + RR_{zp} / R_z| \, |\sum_{j \neq k} \partial p / \partial z^j|.$$

Where recall that m denotes net imports of x. The condition for strategic complements is that the term inside the first absolute value signs on the right side be negative:

$$m\varepsilon_{MD,R} / \Phi + RR_{zp} / R_z < 0. \tag{A3}$$

Note that these two conditions are not inconsistent. For concreteness, consider the case where country k exports x. Then $m < 0$ and $R_{zp} > 0$ (as shown in the text) and (A3) is met if the income response to the terms of trade effect is sufficiently large. Stability requires the income effect not be too large. To see that it is possible for both to hold consider a country with a level of m_0 that satisfies (A3) with equality. Then stability is necessarily met. But now raise exports of the dirty good slightly; that is consider $m_1 < m_0$. Then (A3) is necessarily met and for m_1 close to m_0 both the stability condition and (A3) are satisfied.

Proof of Corollary 3.1

First suppose the country is specialized in X. Then $\varepsilon_{\tau p} = 1$. And since $MD_Z = 0$, then $\varepsilon_{MD,z} = 0$. Plugging these into (C1) and noting that $\theta_X = 1 - \theta_{Xc}$ if the country is specialized in X, reduces (C1) to $\varepsilon_{MD,R} > 1$. If instead the country is specialized in the clean good, then $\varepsilon_{\tau p} = 0$ and $\theta_X = - \theta_{Xc}$. Again, plug into (C1) to obtain $\varepsilon_{MD,R} < 1$. Noting that $m < 0$ if the country is specialized in X and $m > 0$ when specialized in Y yields the result.

Proof of Proposition 4

Given Z, the allocation defined by (20) is unique if G_Z is strictly decreasing in z; that is, if

$$G_{Zp}(dp/dz) + G_{ZZ} < 0. \qquad \text{(A4)}$$

But $G_{ZZ} \leq 0$ since G is concave in z; and $G_{Zp} = \partial\tau/\partial p > 0$ by the Stolper-Samuelson Theorem (both goods are produced in autarky since both are essential). By homotheticity we can let $RD(p)$ denote relative demand (X/Y), and by constant returns to scale we can let $RS(p,z)$ denote relative supply (X/Y). Then in autarky, $RD(p) = RS(p,z)$, and

$$\frac{dp}{dz} = -\frac{RS_z}{RS_p - RD_p} < 0.$$

The inequality follows because (i) by the Rybczinski Theorem, $RS_Z > 0$ since X is pollution intensive, (ii) $RS_p < \infty$ since technology is strictly concave and since X is always strictly more pollution intensive than Y; and (iii) $RD_p < \infty$ since preferences over goods are strictly quasiconcave. Plugging dp/dz into (A4) yields the desired inequality. The second part of Prop. 4 refers to (21) which was derived in the text.

Proof of Proposition 5

For any Z, follow Dixit and Norman (1980) and consider the hypothetical equilibrium that would obtain if there were an integrated world economy with free factor mobility across countries and with endowment H=h+h* and Z. Let (w^o,τ^o) and (x^o,y^o) be the equilibrium factor price and world output vectors in this integrated equilibrium, and let a_x and a_y be the corresponding unit input vectors for x and y ($a_x \equiv (h_x,z_x)$ etc.). Now consider the set

$$V = \{(h,z) \mid \exists\, (x,y) \text{ s.t. } 0 \le x \le x^o,\ 0 \le y \le y^o, \text{ and } xa_x + ya_y = (h,z)\}.$$

This is the set of allocations of endowments to the West for which it is possible to solve West's full employment conditions at factor prices (w^o,τ^o) with outputs no larger than produced in the integrated equilibrium. If this can be done, then East's full employment conditions are also automatically satisfied (let z* = Z - z, $x^* = x^o - x$, and $y^* = y^o - y$). Because preferences over goods are identical and homothetic, relative demand is unaffected. These allocations therefore yield free trade equilibria which replicate the integrated equilibrium, and hence in which permit prices are the same in West and East.

The set V defines a non-degenerate parallelogram since the vectors a_x and a_y are linearly independent (since x is strictly more pollution intensive than y). Now consider West's h. This yields a slice through the parallelogram V; that is, for any h, there is a continuum of z in V. Hence for any global pollution level Z, there are infinitely many allocations z and z* which replicate the integrated equilibrium and which are therefore efficient.

Proof of Proposition 6

Since both countries are initially diversified, each country's endowment ratio must lie between the equilibrium industry input ratios. Hence $z_y/h_y < z/h < z_x/h_x$ in the West and $z_y/h_y < z^*/h^* < z_x/h_x$ in the East. With equiproportionate reductions, we have $\hat{z} = \hat{z}^* = \hat{Z}$. Since h and h* are given, factor price equalization will continue to hold if $(\hat{z}_x / h_x) = (\hat{z}_y / h_y) = \hat{Z}$ and similarly for the East. But with standard manipulations (see Jones (1965)), we have:

$$(\hat{z_i / h_i}) = \frac{\sigma_i \hat{Z}}{|\theta||\lambda|(\sigma_s + \sigma_D)}$$

for i = x,y, where σ_i is the elasticity of substitution in industry i, σ_S is the elasticity of substitution between X and Y along the production frontier, σ_D is the elasticity of substitution in demand, $|\theta| = \theta_{XZ} - \theta_{yZ} > 0$, $|\lambda| = \lambda_{XZ} - \lambda_{XL} > 0$, θ_{ij} is the share of input j in the cost of good i, and λ_{ij} is the fraction of input j employed in sector i. If $\sigma_X = \sigma_y = \sigma_D = 1$, the above simplifies to $(\hat{z_x / h_x}) = (\hat{z_y / h_y}) = \hat{Z}$, as required for the result.

Proof of Proposition 7

Suppose there is free trade in emission permits but no goods trade. West's income is:

$$I = G(p, h, z + z^I) - \tau^* z^I$$

where z^I denotes Western net imports of pollution permits and τ^* is the world price of permits. Suppose that West decides to change z^I. Differentiating (4), with world Z fixed, and starting from the point where there is free trade in the permit market, yields:

$$\left.\frac{du}{dz^I}\right|_{\tau=\tau^*} = -\frac{u_R}{\Phi} z^I \frac{d\tau^*}{dz^I}. \qquad \text{(A6)}$$

Since Eastern permit demand is downward sloping, we can show $d\tau^*/dz^I > 0$. If West is a permit importer ($z^I > 0$), then (A6) is negative and West can increase its welfare by restricting permit imports. If West is a permit seller ($z^I < 0$), then (A6) is positive and West has an incentive to reduce its permit exports.

Now suppose there is free trade in goods and the permit allocation is in the interior of $S^A S^C$ in Fig. 3. Equilibrium can be achieved via either permit trade, goods trade, or some combination of both. Suppose West attempts to manipulate the permit market with a small change in z^I. Since the East's permit supply curve is perfectly elastic in this range, any small change in z^I has no effect on τ^*. And moreover, since permit prices do not change, goods prices will not change either. Hence starting in $S^A S^C$, we have $du/dz^I = 0$.

Proof of Proposition 8

This is a standard gains-from-trade proof (see Grossman, 1984). Let p be the goods price vector after free permit trade, let τ be the equilibrium permit price and let z^I be net imports of permits. Also u is utility after permit trade, u^a is utility prior to permit trade, and x^a and y^a are outputs prior to permit trade. Let E(p,Z,u) be the expenditure function. Then:

$$E(p,Z,u) = G(p,z+z^I) - \tau z^I \geq px^a + y^a \geq E(p,Z,u^a)$$

The first inequality follows since the private sector maximizes national income: pre-permit trade outputs (x^a,y^a) are feasible but not optimal after permit trade. The next inequality follows since (x^a,y^a) yields utility u^a (because there is no goods trade), but this utility could be attained at lower cost given the new prices p. Finally, $u \geq u^a$ since E is increasing in u.

Proof of Proposition 9

The proof is by example. Suppose u = xy – D(Z). Assume endowments are mirror images (z* = h, h* = z); and East is abundant in permits (z*/h* > z/h). Let technology be $x = z^{\beta}h^{1-\beta}$ and $y = z^{1-\beta}h^{\beta}$, with $\beta > 1/2$ so that x is pollution intensive. By symmetry, p = 1 in free trade. The boundaries of the cone of diversification are $z_x/h_x = \beta/(1-\beta)$ and $z_y/h_y = (1-\beta)/\beta$. Suppose $z^*/h^* > \beta/(1-\beta)$ and $z/h < (1-\beta)/\beta$, so that both countries specialize in production. Because permits are scarce in the West, we have $\tau > \tau^*$.

Suppose West imports a permit at the Eastern price τ^*; that is, West receives all of the direct gains from trade. The effect on West's welfare is

$$E_u du = (\tau - \tau^*)dz^I - m dp. \qquad \text{(A7)}$$

Imports m are just West's demand for X, and hence m = Y/2p (West's income is just Y). To find dp, equate relative demand and supply to obtain p = Y/X*. Differentiating and using the condition that the value of the marginal product of emissions is the permit price, we obtain $dp = (p\tau/Y + \tau^*/X)dz^I$. Substituting for dp into (A7) and simplifying yields

$$E_u du = (\tau - 3\tau^*)/2. \tag{A8}$$

But from cost minimization, $\tau z/Y = (1-\beta)$ and $\tau^* z^*/X^* = \beta$. By symmetry, Y=X*, and so $\tau/\tau^* = \beta z^*/(1-\beta)z$. Using this in (A8) shows that if

$$\frac{\beta}{1-\beta} < \frac{z^*}{z} < \frac{3(1-\beta)}{\beta}.$$

then we have both $\tau > \tau^*$ and $E_u du/dz^I < 0$. Finally note that for this to be possible and consistent with X being pollution intensive, we require $\beta \in (1/2, 3/4)$.

Proof of Proposition 10

To show that both West and East can lose from permit trade in a three-region world, it is sufficient to consider the case where $\varepsilon^S_{MD,R} = 1$, so that Southern pollution does not change. Totally differentiating the market clearing condition for X yields

$$\frac{dp}{dz^I} = \frac{x_I(\tau^W - \tau^E) - (\frac{\partial x^W}{\partial z^W} - \frac{\partial x^W}{\partial z^W})}{D}, \tag{A9}$$

where $x_I = \partial x^c/\partial I$ (where x^c is the demand for X in the North) and I is Northern income), $D = H + x_I m + x_I m^S$, with $H = G^W_{pp} + G^E_{pp} + G^S_{pp} - E_{pp} - E^S_{pp} > 0$, and where m denotes Northern (aggregate East and West) imports of X and m^S Southern net X imports. Stability requires D > 0. Using (A9) in (23) (but with $dz^S/dp = 0$) yields

$$\left.\frac{du}{dz^I}\right|_{z^I=0} = \frac{u_R}{\Phi(p)}\left[\frac{(\tau^W - \tau^E)(H + x_I^S m^S) - m\,\frac{\partial x^E}{\partial z^E}}{D}\right],$$

where $m^S < 0$ since South exports X and $\partial x^E/\partial z^E > 0$ since X is intensive in emissions. There are two ways that utility may fall from the permit trade. First, if pure substitution effects (embodied in H) are locally small, and the Southern income effect ($x_I^S m^S < 0$) is large, then utility may fall. This corresponds to the case of an inelastic foreign offer curve, since if the foreign income effect dominates the substitution effect, an increase in the price of X leads to a fall in foreign exports. Second, even if this condition is not satisfied, a strong fall in X production in East can be enough to cause a price increase big enough to lower Western utility. This shows that it is possible for the terms of trade loss from permit trade to be larger than the direct permit trade gains in three country context. Hence West and East can be collectively worse off, and there will exist some division of the permit trade gains such that both can lose from the trade. Finally, the result that West-East trades can affect Southern pollution follows from (24).